Composite Implied Volatility and Dynamic Selection Models in Pricing and Hedging Options

Chuang, I-Yuan

Department of Finance, National Chung Cheng University 168 University Road, Min-Hsiung, Chiayi 621, Taiwan E-mail: finiyc@ccu.edu.tw Tel: +886-(0)5-272-0411 ext.34211

Lee, Pei-Hsuan

Corresponding Author, Department of Finance National Chung Cheng University, 168 University Road Min-Hsiung, Chiayi 621, Taiwan E-mail: patricia@ccu.edu.tw Tel: +886-(0)5-272-0411 ext.34208

Tsai, Tsung-Min

Department of Finance, National Chung Cheng University 168 University Road, Min-Hsiung, Chiayi 621, Taiwan E-mail: tsungmin2001@yahoo.com.tw

Abstract

This research compares a variety of implied volatility specifications incorporated into the Black model for the pricing and hedging of the TAIEX option market. The specific aim of the research is to test the composite volatility and dynamic selection specifications. The former contains three methods for calculating composite volatility: the simple, the minimum error-weighted average and the geometric weighted average. The latter selects, day by day, the volatility model which outperforms others to carry out forecasting for the next day. The result indicates that the model-free specification ranks the first as the pricing option, and the composite volatility second, while the dynamic selection model is not prominent. Regarding hedging performances, the composite volatility is outstanding compared to the others, second is the model-free specification and the dynamic selection model falls behind.

Keywords: Composite Forecasting; Index Option; Implied Volatility; Option Pricing; Option Hedging

1. Introduction

On December 24, 2001, the Taiwan Stock Exchange Capitalization Weighted Stock Index Options Contracts (TAIEX Options) started trading on the Taiwan Futures Exchange; it was the first time options had been traded in Taiwan. At first, the daily average trading volume was only 856 contracts in

2001; the market grew fast, and the daily average trading volume increased to 372,519 contracts in 2008.

Based on the no-arbitrage condition and dynamic hedging with underlying assets and cash, Black and Scholes (1973) introduced the famous Black-Scholes option pricing model. There are six parameters in this model: the current price of the underlying asset, strike price, risk-free rate, volatility, time to expiration and dividend rate. With the exception of volatility, the other parameters can be obtained by observing the data in the market; therefore, the estimation of volatility is of great importance in terms of pricing and hedging. Previous studies estimated the volatility in two ways: firstly, the historic volatility based on past trading information of the underlying assets, and secondly, the implied volatility based on the cross-section market data. This study focuses on implied volatility models. Theoretically, options with the same underlying assets are supposed to have the same implied volatility; however in practice, different implied volatilities for options with the same underlying assets may be obtained due to different expiration dates or features. As a result, previous literature shows that various weighted average methods were developed for implied volatilities. Besides adopting the simplest equally-weighted average implied volatility, the study also refers to the Vega-weighted average by Latane and Rendleman (1976), the elasticity-weighted average by Chiras and Manaster (1978), the volume-weighted average by Day and Lewis (1988) and the least square model by Beckers (1981) to calculate the implied volatility.

The calculation of Black-Scholes implied volatility involves specific pricing models. However, model assumptions are not necessarily consistent with actual practice. Hence, in addition to the Black-Scholes implied volatility model, this study also uses the volatility index (VIX) and model-free implied volatility models, both of which do not involve any pricing models. VIX was introduced by the Chicago Board Options Exchange (CBOE) in 1993. On September 22, 2003, the CBOE began disseminating price level information using a revised methodology for the CBOE VIX; a series of index options with different exercise prices was used for the implied volatility, but the formula did not involve any pricing models in order to fully reflect the market dynamics. In addition, countries such as France, Germany and Switzerland have developed VIX for their own equity markets as a reference for short-term volatility indices across different countries and used the simulated data of the TAIEX index options to construct the VIX for TAIEX and test its forecasting power. Their results suggest that VIX is a good estimator of future volatility and a good contrarian trading indicator when the market plunges.

Britten-Jones and Neuberger (2000) deduced a model-free implied volatility that is the expected sum of squared returns under a risk neutral measure. The only criterion for model-free implied volatility is no arbitrage, and the formula does not involve any option pricing model. However, the model-free implied volatility in Britten-Jones and Neuberger (2000) does not include potential asset jumps. Jiang and Tian (2005) extended the model to include asset jumps and used observed option prices to calculate the implied volatility.

A huge number of studies on volatility models have been conducted, many of which focus on the predictive power or information content of implied volatility, such as Latane and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981), Gemmill (1986), Day and Lewis (1992, 1993), Canina and Figlewski (1993), Christensen and Prabhala (1998), Gwilym and Buckle (1999), Christensen and Hansen (2002), and Szakmary et al. (2003). The studies compared the predictive power of implied volatility and historic volatility on realized volatility, prediction error and the bias of information content; different results were yielded. Furthermore, Chuang et al. (2009) took TAIEX Options as a case study to compare the mean absolute error and root mean square error, using 15 volatility models: the common volatility models used in previous studies, the VIX implied volatility, model-free implied volatility, the Corrado & Su and SGT implied volatility and the volatility model containing intraday information, as specified in Parkinson (1980) and Yang and Zhang (2000). Chuang et al. (2009) discovered that with the exception of the EGARCH volatility, the predication error under

historic volatility is larger than it is under the implied volatility. As for option pricing and hedging, Chuang et al. (2009) also found that implied volatility models perform better in general.

The Black-Scholes model, a benchmark model in option pricing, is also widely used in the industry. Black (1990) made the following comments:

"I sometimes wonder why people still use the Black and Scholes formula, since it is based on such simple assumptions—unrealistically simple assumptions. Yet that weakness is also its greatest strength. People like the model because they can easily understand its assumptions."

On the other hand, previous studies show that on average, the implied volatility performs better in prediction, pricing and hedging. For this reason, we decided to study implied volatility models further.

The study compares pricing and hedging performance in different implied volatility models applicable to the Black-Scholes model, and discusses reasons for bias. In particular, this study also introduces composite volatility and dynamic selection approach to assess their applications in option pricing and hedging performances. The concept of composite prediction is based on Bates and Granger (1969), and extended by Granger and Ramanathan (1984). We integrated these two models in the hope that they will improve volatility predictions and thereby reduce pricing and hedging errors. Ederington and Guan (2002) studied the composite implied volatility; however, they only made predictions on realized volatility without examining pricing or hedging performance. We decided to study these issues further. Under dynamic selection, the best available volatility model was selected at each prediction date. In other words, the model changes for each day, so it is called the dynamic selection model. To our knowledge, this is the first study to apply the dynamic selection approach on options research.

The paper is organized as follows: firstly, the introduction; secondly, our methodology; thirdly, the data processing; fourthly, the empirical analysis; and finally, the conclusion.

2. Methodology

There are four implied volatility models used in the study: Black implied volatility (with different weighting schemes), composite implied volatility, model-free implied volatility and dynamic selection implied volatility. In this section, we introduce these implied volatility models, followed by measurements for option pricing and hedging performance. Finally, we discuss the error analysis.

2.1. Black Model and Black Implied Volatility

The implied volatility is solved using the price of an option working with the option pricing model in reverse; the estimation is based on the pricing model. The Black-Schole Model or Black Model is used in most studies.

TAIEX Options and TAIEX spot are traded in different markets, and the option market closes 15 minutes later than the spot market does; therefore, a problem of non-synchronous trading exists. Second, the TAIEX spot is a basket of securities; it is necessary to replicate the TAIEX spot in order to arbitrage, which is costly and makes arbitrage difficult. Third, it is necessary to estimate the dividend rate of the index when applying the Black-Scholes model to option pricing; however, making an accurate estimation is difficult and challenging.

Previous studies often solve the aforementioned three issues using futures contracts. First, the futures price reflects overall market conditions. In estimating the volatility, the implied volatility based on the futures price integrates all variables that affect stock volatility; it avoids the non-synchronous problem of index price (Feinstein, 1989) and sampling (Hwang and Satchell, 2000), since the closing index does not necessarily reflect the closing price of all index components. Second, to build a portfolio of a basket of stocks replicating the index option is costly; therefore, the market maker for index options tends to use futures instead of spot in hedging. Lastly, the futures price reflects the carrying costs; it includes the dividend rate. Hence it is no longer necessary to estimate the dividend rate (Duan and Zhang, 2001). As a result, the Black model is often chosen in empirical research

(Jorion, 1995; Hwang and Satchell, 2000; Duan and Zhang, 2001). This study replaces the TAIEX spot price with the TAIEX futures price. The Black Model is as follows:

$$C = e^{-r\tau} FN(d_1) - e^{-r\tau} KN(d_2)$$
(1)

$$P = e^{-r\tau} KN(-d_2) - e^{-r\tau} FN(-d_1)$$

$$d_1 = \frac{\ln(F/K) + \tau \sigma^2 / 2}{\sigma \sqrt{\tau}}$$

$$d_2 = d_1 - \sigma \sqrt{\tau}$$

where *C* is the call price; *P* is the put price; *F* is the futures price; *K* is the strike price; τ is the time to maturity; *r* is the risk-free rate; σ is the volatility; and *N*(•) is the cumulative probability density function of the standard normal distribution.

Equally-Weighted Average Implied Volatility

Having obtained the implied volatility for each option contract, we can use the equally-weighted average to assess the representative implied volatility so that each contract is covered in the calculation, as the following equation shows:

$$\sigma_{EW,t} = (1/k) \sum_{j=1}^{k} IV_j$$
(2)

where $\sigma_{EW,t}$ is the equally-weighted average volatility at time t; j = 1, 2, ..., k, and k is the number of options observed daily; IV_j is the implied volatility of the j^{th} TAIEX option on the same observation day.

Vega-Weighted Average Implied Volatility

Options at different price levels are not equally sensitive to the volatility; hence, it is inappropriate to allocate the same weight. Latane and Rendleman (1976) suggested the Vega-weighted average, where an implied volatility is given a larger weight if the option price is more sensitive to the volatility. Since at-the-money options have the largest Vega values, they are given larger weights. The following is the formula for the Vega-weighted average:

$$\sigma_{Vega,t} = \sum_{j=1}^{k} (IV_j D_j) / \sum_{j=1}^{k} D_j$$
(3)

where $\sigma_{Vega,t}$ is the Vega-weighted average volatility; $D_j = \partial V_j / \partial \sigma_j$ is the Vega value of the *j*th TAIEX option contract on the same observation day; *V* is the call or put price; and *k* and *IV_j* are defined as before.

Elasticity-Weighted Average Implied Volatility

Chiras and Manaster (1978) weighted the volatility according to its elasticity to the option price. This reflects options sensitivities at different price levels. The following is the formula for elasticity-weighted volatility:

$$\sigma_{ELS,t} = \sum_{j=1}^{k} (IV_j E_j) / (\sum_{j=1}^{k} E_j)$$
(4)

where $\sigma_{ELS,t}$ is the elasticity-weighted average implied volatility; E_j is the price elasticity of the volatility, which is $(\partial V_j / \partial \sigma_j)(\sigma_j / V_j)$; and V_j , k and IV_j are defined as before.

Volume-Weighted Average Implied Volatility

The trading volume indicates market efficiency, reflecting market volatility. As a result, Day and Lewis (1988) weighted the volatility based on the daily trading volume, giving options with a larger volume a larger weight. The formula is as follows:

$$\sigma_{VW,t} = \sum_{j=1}^{k} IV_j \cdot TV_j / \sum_{j=1}^{k} TV_j$$
(5)

where σ_{VW} is the volume-weighted average of the implied volatility; TV_j is the trading volume of the j^{th} TAIEX option on the same observation day; and V_j , k and IV_j are defined as before.

Least-Square Implied Volatility

Options at different price levels are not equally sensitive to price fluctuations; therefore, they also have different explanatory powers for the volatility. The Vega-weighted average reflects the sensitivity of at-the-money options and reduces volatility smile. Beckers (1981) combined the least square error method and Vega-weighted average to formulate the volatility:

$$\underset{\sigma_{MW}}{Min}\sum_{j=1}^{k} Vega_{j}[V_{M,j} - V_{BL,j}(\sigma_{MW})]^{2}$$
(6)

where σ_{MW} is the least-square volatility; $V_{M,j}$ is the market price of the j^{th} option; and $V_{Bl,j}(\sigma_{MW})$ is the theoretical option price given σ_{MW} , under the Black model.

2.2. Composite Implied Volatility

We applied the following three averages for calculating composite implied volatility: the simple average, the minimum error-weighted average and the geometric average. First, we introduced the simple average implied volatility:

$$\sigma_{CEW,t} = (\sigma_{EW,t} + \sigma_{Vega,t} + \sigma_{ELS,t} + \sigma_{VW,t} + \sigma_{MW,t})/5$$
(7)

Specifically, this composite implied volatility is the simple average of the aforementioned implied volatilities. The weights of the minimum error-weighted average are assigned according to the minimum mean squared error criterion. That is, the minimum error-weighted average estimates the daily model deviation, and then different weights are assigned according to their deviations. Suppose the implied volatility σ_i of model *i* is calculated daily. Meanwhile, the root mean squared error (RMSE) under the given model on that day is defined as:

$$\varepsilon_i = \sqrt{\frac{1}{k} \sum_{j=1}^k [V_{M,j} - V_{BL,j}(\sigma_i)]^2}$$
(8)

where *k* is the number of options observed every day; $V_{M,j}$ is the market price of the *j*th option; and $V_{BL,j}(\sigma_i)$ is the theoretical price under Black model given σ_i . Granger and Ramanathan (1984) suggested the weight for model *i* to be:

$$\mathbf{w}_{i} = \frac{\frac{1}{\varepsilon_{i}}}{\frac{1}{\varepsilon_{1}} + \frac{1}{\varepsilon_{2}} + \dots + \frac{1}{\varepsilon_{5}}}$$
(9)

where ε_i is the RMSE of the *i*th model. According to the formula, models with a smaller RMSE have larger weights. The minimum error-weighted average σ_{CMW} is:

$$\sigma_{CMW,t} = w_1 \cdot \sigma_{EW,t} + w_2 \cdot \sigma_{Vega,t} + w_3 \cdot \sigma_{ELS,t} + w_4 \cdot \sigma_{VW,t} + w_5 \cdot \sigma_{MW,t}$$
(10)
Let us introduce the geometric average volatility as below:

Lastly, we introduce the geometric average volatility as below:

$$\sigma_{CGW,t} = (\sigma_{EW,t} \cdot \sigma_{Vega,t} \cdot \sigma_{ELS,t} \cdot \sigma_{VW,t} \cdot \sigma_{MW,t})^{1/5}$$
(11)

2.3. Model-Free Volatility

Individual implied volatility models all require a financial model to perform volatility estimation. However, assumptions in the model often deviate from the practice, which implies errors. Model-free volatility, by avoiding modeling estimation errors, may better reflect market dynamics. This study used the VIX volatility and model-free implied volatility as specified below.

VIX Implied Volatility

To revise VIX, the Chicago Board Options Exchange (CBOE) used a series of index options with different strike prices to calculate the implied volatility. The calculation does not involve any pricing models, as in the following formula:

$$\sigma_{VIX,t} = \frac{2}{T} \sum_{i} \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2$$
(12)

where $\sigma_{VIX,t}$ is VIX's volatility; *F* is the expected index derived from the option price; *T* is the duration; K_i is the strike price for the *i*th out-of-money option; we used the call price if $K_i > F$ and put price if $K_i < F$; $f: \triangle K_i = 0.5 \times (K_{i+1} - K_{i-1})$; K_0 represents the first strike price lower than the expected index *F*; *r* is the risk-free rate; $Q(K_i)$ is the average of the bid and ask given to an option with a strike price of K_i .

Model-free Implied Volatility

Jiang and Tian (2005) extended the model-free implied volatility in Britten-Jones and Neuberger (2000) to asset price processes with jumps and developed a simple method for implementing it using observed option prices. Such volatility is solely derived from the no-arbitrage condition and involves no pricing models. We used the numerical methods in Jiang and Tian (2005) to calculate the model-free implied volatility:

$$\sigma_{MF}^2 = 2 \times \int_0^\infty \frac{C(\tau, K) - \max(0, F - K)}{K^2} dK$$
(13)

where σ_{MF} is the model-free implied volatility; τ is the time to maturity; *F* is the futures price; and *K* is the strike price; $C(\tau, K)$ is the option price.

2.4. Dynamic Selection Implied Volatility

The dynamic selection implied volatility model searches for the minimum RMSE (see Equation (8)) among the aforementioned five Black implied volatility models, three composite implied volatility models and two model-free volatility models. Therefore, different models are used in different periods. We selected the model with minimum root mean square error to represent the current implied volatility, that is:

 $\sigma_{DS} = \sigma$ with minimum ε defined by Equation (8)

2.5. Option Pricing and Hedging Performance

We adopted the out-of-sample performance evaluation in this research. First, the data on the t^{th} day were used to estimate the implied volatility, and then we applied the estimated implied volatility to the option pricing model for the theoretical option price on the $t+1^{\text{th}}$ day. Next, we compared the theoretical and market price of the option on day t+1, calculated the errors and evaluated the out-of-sample predictability. We used MAE and RMSE to determine the model fitness for performance evaluation. The equation for the performance evaluation is as follows:

$$MAE = \frac{1}{N} \sum_{j=1}^{N} \left| V_{M,j} - V_{BL,j} \right|$$
(14)

$$RMSE = \frac{1}{N} \sum_{j=1}^{N} \sqrt{(V_{M,j} - V_{BL,j})^2}$$
(15)

N is the total number of options; other symbols follow the same definitions as in the previous session.

The study adopted the hedge model in Dumas, Fleming and Whaley (1998). Assuming that option returns can be replicated by continuously adjusting the portfolio, and ignoring the restrictions on trading time and trading costs, we analyzed the hedge error under each model. Let h be the hedge ratio. According to the definition in Dumas, Fleming and Whaley (1998), the hedging error can be shown as:

$$e_t = \Delta C_{actual,t} - \int_t^{t+n} h(S_u, u) dS_u$$
(16)

where e_t is the hedging error; $\Delta C_{actual,t}$ is the actual change in option price; and *n* is the number of hedging days. Dumas, Fleming and Whaley (1998) further expressed the hedging error, after deduction, in the following equation:

$$e_t = \Delta C_{actual,t} - \Delta C_{\text{mod}\,el,t} \tag{17}$$

where $\Delta C_{model,t}$ represents changes in the theoretical option price. Assuming we used the correct model to determine the hedge ratio *h*, then the expected value and variance for the above hedging error e_t are supposed to be zero. We used Equation (17) to construct the MAE and RMSE for hedging performance.

2.6. Error Analysis

We then applied regression analysis to the error term; this will be a reference to future pricing or hedging. For each contract, let the absolute error $AE = |V_M - V_{BL}|$ be the dependent variable, and the price F/K, expiration (τ), risk-free rate (r), trading volume (TV), open interests (OI), and put-call dummy variable (CP) be the independent variables in the following regression:

$$AE = \beta_0 + \beta_1 F/K + \beta_2 \tau + \beta_3 r + \beta_4 TV + \beta_5 OI + \beta_6 CP + e$$
(18)
The same analysis can also be applied to the hedging error.

3. Data Processing

The trading volume of TAIEX Options in the Taiwan Stock Exchange was significantly lower at first. The market was not as mature and efficient; there may have only been a few transactions for in-themoney or out-of-the-money options. However, since October 2002, the daily average volume of TAIEX options has been at least 10,000 contracts, and the market has become more mature. This study sampled 89,590 contracts from January 1, 2003 to June 30, 2009; the data includes: TAIEX options, TAIEX futures, TAIEX and the risk-free rate, where the data for options and futures were from the Taiwan Futures Exchange, and the data for TAIEX and risk-free rate were from the Taiwan Economic Journal Database (TEJ).

The option and futures contracts differ in expiration terms, so we selected futures to accommodate the expiration date of options. Foreign studies often use the following rates as the risk-free rate: short-term government bonds, short-term treasury bills or the London Interbank Offered Rate (LIBOR) with the mature day the same as, or similar to, the expiration of the option. Domestic studies often use the bank-announced term deposit rates (for example, the Bank of Taiwan, First Commercial Bank, Hua Nan Commercial Bank and Chang Hwa Commercial Bank).

This study selected the risk-free rates that have a maturity the same as, or similar to, the expiration of the option; the benchmark for the risk-free rate is the 1-month, 3-month, 6-month, 9-month, and 1-year term deposit rates announced by the Bank of Taiwan. The following is how we selected the rate: if the time to option expiration ranges from 1 to 40 days, the 1-month term deposit rate applies; if it is 41 to 70 days, the 3-month term deposit rate applies; if 71 to 150 days, the 6-month term deposit rate applies; if 151 to 210 days, the 9-month term deposit rate applies; if 211 days or more, the 1-year term deposit rate applies. The term deposit rates announced are annual percentage rates; therefore we first converted the rates into continuous compounding rates for model specification.

Next, we used the following criteria to select the data: the price level of options, trading volume, no-arbitrage condition and time to expiration. First, the option price cannot be lower than 3 points. If an option price is too low, it may be affected by the bid-ask spread and trading costs; such options are often removed in the studies, for example Schmalensee and Trippi (1978), Feinstein (1989), and Donders and Vorst (1996). The Taiwan Futures Exchange regulates that, besides the premium and margin, investors are supposed to pay fees and transaction tax in option transactions;

either party (buyer or seller) pays the transaction tax; i.e., the transaction tax is imposed on option buyers and sellers. The tax rate is 1.25 thousandth of the premium before the expiration and 0.25 thousandth of the settlement value at the expiration. The fee standard is determined by securities firms; in general, the fee is NT\$ 128 per contract. As a result, option prices below 3 points are excluded from our dataset.

Second, the volume cannot be less than 10 contracts. If an option is not traded on a certain day, or only has a small volume traded by the market maker, these transactions do not reflect market conditions. In reference to George and Longstaff (1993), we filtered options with a volume of less than 10.

Third, the option price cannot fall below the lower bound of arbitrage. If the price is below the lower bound of risk-free arbitrage, the market is inefficient. We screened out such data to avoid unreasonably implied volatilities.

Fourth, the remaining time to expiration cannot be less than 5 business days. As the expiration date approaches, the option price is more likely to be affected by the price of the underlying asset, creating higher price volatility. Thus, we removed the data if the remaining time to expiration was less than 5 business days.

4. Empirical Results

When comparing option pricing and hedging errors, we evaluated the performance of the full sample, the call options, and put options. In addition, according to moneyness, options (call option: F/K; put option: K/F) were classified as at the money (ATM, 0.97~1.03), in the money (ITM, 1.03~1.1), and out of the money (OTM, 0.9~0.97). The mean absolute error (MAE) and the root mean squared error (RMSE) were used to measure pricing and hedging performances in all implied volatility models. We also investigated important factors resulting in the difference between pricing and hedging performances.

4.1. Pricing Performances of TAIEX Options

Table 1 shows the pricing performances for all out-of-sample models. The volatility model which best reflected the option prices for the call option, put option and options with different moneyness was thus determined. This result indicated that the model-free implied volatility model (BK_MF) had the best pricing performance across the samples and the subsamples of call options and put options. The three composite volatility models came in second. The dynamic selection model (BK_DS) did not yield a better performance. Of the three composite volatility models, the geometric average volatility model (BK_CGW) had the smallest pricing error, followed by the minimum error-weighted average model (BK_CMW). However, there were no significant differences among the three composite volatility models.

With different moneyness, the out-of-sample performances were identical to full-sample results; that is, regardless of ITM, ATM or OTM, the model-free implied volatility model had the best performance, followed by the composite volatility models. We also found that OTM options had smaller pricing errors than ITM and ATM options probably because OTM options have lower prices, causing the absolute error to be smaller.

Overall, the model-free model had the best out-of-sample pricing performance, followed by the composite volatility models. The model-free implied volatility models (i.e., BK_MF and BK_VIX) had a better pricing performance regardless of the call option, put option or different moneyness. On the contrary, the VIX volatility model did not have such a performance. As for the composite volatility models, the geometric weighted volatility model (BK_CGW) yielded the smallest pricing errors; the minimum error-weighted average model (BK_CMW) came in second. However, the dynamic selection model (BK_DS) did not significantly outperform other models.

		Whole sample		Out-of-t	Out-of-the-money		At-the-money		money
		MAE	RMSE	MAE	RMŠE	MAE	RMSE	MAE	RMSE
	BK_EW	7.37 (8)	11.24 (6)	6.78 (10)	10.46 (6)	8.27 (10)	11.65 (7)	7.97 (8)	12.19 (7)
	BK_Vega	8.32 (11)	15.98 (11)	7.35 (9)	14.55 (11)	8.20 (9)	14.76 (11)	8.81 (11)	15.74 (11)
	BK_ELS	8.16 (10)	12.39 (10)	7.74 (11)	11.90 (10)	9.73 (11)	13.54 (10)	8.71 (10)	13.08 (10)
Whole	BK_VW	6.96 (5)	11.40(7)	6.51 (1)	10.77 (7)	6.68(1)	11.07 (5)	7.62 (4)	12.08 (5)
	BK_MW	7.03 (6)	10.76 (5)	6.37 (7)	9.70 (5)	7.53 (7)	11.10(6)	7.95 (6)	12.12 (6)
sample	BK_CEW	6.83 (4)	10.58 (4)	6.21 (5)	9.68 (4)	7.24 (5)	10.62 (4)	7.63 (5)	11.76 (4)
	BK CMW	6.81 (3)	10.57 (3)	6.20 (4)	9.67 (3)	7.20 (4)	10.59 (3)	7.62 (3)	11.75 (3)
	BK CGW	6.80(2)	10.56 (2)	6.19 (3)	9.65 (2)	7.19 (3)	10.58 (2)	7.61 (2)	11.74 (2)
	BKVIX	7.64 (9)	10.26 (9)	7.05 (8)	11.47 (9)	8.13 (8)	12.72 (9)	8.31 (9)	12.99 (9)
	BKMF	6.70(1)	10.42 (1)	6.03 (2)	9.43 (1)	6.94 (2)	10.29(1)	7.57(1)	11.71 (1)
	BK_DS	7.26(7)	11.59 (8)	6.78 (6)	10.82 (8)	7.39 (6)	11.75 (8)	7.96 (7)	12.34 (8)
	BK_EW	7.70 (8)	11.46 (7)	8.46 (9)	12.15 (8)	7.98 (8)	11.26(7)	6.43 (5)	10.24 (5)
	BK_Vega	9.21 (11)	13.12 (11)	9.66 (11)	13.79 (11)	8.06 (9)	14.26 (11)	6.50 (6)	10.95 (6)
	BK_ELS	8.47 (10)	12.63 (9)	9.37 (10)	13.67 (10)	9.44 (11)	13.20 (10)	7.20 (9)	11.14 (9)
	BK_VW	7.07 (6)	11.53 (8)	7.08 (3)	11.59 (7)	6.63 (1)	11.02 (6)	6.89 (7)	11.00(7)
Call	BK_MW	7.05 (5)	10.67 (5)	7.06 (2)	10.38 (1)	7.31 (6)	10.70 (5)	7.16 (8)	11.09 (8)
	BK_CEW	7.02 (4)	10.67 (4)	7.46 (7)	10.97 (6)	6.97 (5)	10.24 (4)	6.40 (3)	10.21 (2)
	BK_CMW	7.00 (3)	10.65 (3)	7.43 (6)	10.94 (5)	6.93 (4)	10.21 (3)	6.40 (2)	10.21 (3)
	BK_CGW	6.99 (2)	10.63 (2)	7.41 (5)	10.91 (4)	6.92 (3)	10.20 (2)	6.40 (4)	10.22 (4)
	BK_VIX	8.03 (9)	12.65 (10)	8.16 (8)	12.79 (9)	8.16 (10)	12.68 (9)	7.24 (10)	11.38 (10)
	BK_MF	6.90 (1)	10.53 (1)	7.27 (4)	10.73 (2)	6.67 (2)	9.90 (1)	6.37 (1)	10.18 (1)
	BK_DS	7.14 (7)	11.39 (6)	6.75 (1)	10.83 (3)	7.38 (7)	11.70 (8)	7.54 (11)	11.57 (11)
	BK_EW	7.03 (7)	11.00 (6)	5.15 (6)	8.49 (5)	8.57 (10)	12.04 (8)	9.83 (8)	14.18 (8)
	BK_Vega	7.41 (10)	12.72 (11)	5.09 (5)	11.10 (11)	8.34 (9)	14.26 (11)	11.59 (11)	15.12 (11)
	BK_ELS	7.84 (11)	12.14 (10)	6.15 (10)	9.88 (7)	10.04 (11)	13.88 (10)	10.54 (10)	15.08 (10)
	BK_VW	6.85 (5)	11.27 (7)	5.96 (8)	9.90 (8)	6.73 (1)	11.13 (5)	8.51 (5)	13.26 (2)
	BK_MW	7.01 (6)	10.86 (5)	5.71 (7)	8.99 (6)	7.76 (7)	11.50 (6)	8.89 (6)	13.26 (3)
Put	BK_CEW	6.63 (4)	10.50 (4)	5.00 (4)	8.24 (3)	7.51 (6)	11.00 (4)	9.10 (4)	13.38 (7)
	BK_CMW	6.62 (3)	10.48 (3)	4.99 (2)	8.24 (2)	7.47 (5)	10.97 (3)	9.08 (3)	13.36 (6)
	BK_CGW	6.62 (2)	10.48 (2)	5.00 (3)	8.25 (4)	7.46 (4)	10.95 (2)	9.06 (2)	13.34 (5)
	BK_VIX	7.25 (8)	11.84 (9)	5.97 (9)	10.02 (9)	8.09 (8)	12.76 (9)	9.60 (9)	14.69 (9)
	BK_MF	6.49 (1)	10.31 (1)	4.83 (1)	7.97 (1)	7.21 (2)	10.68 (1)	9.00 (1)	13.32 (4)
NT - 4	BK_DS	7.39 (9)	11.78 (8)	46.80 (11)	10.80 (10)	7.40 (3)	11.79 (7)	8.46 (7)	13.21 (1)

 Table 1:
 Out-of-the-sample Pricing Performance

Note: EW= equal weighted vol., Vega = Vega weighted vol., ELS = elasticity weighted vol., VW = volume weighted vol., MW = min mean squared errors vol., CEW = composite simple weighted vol., CMW = composite min error-weighted average vol., CGW = composite geometric weighted vol., VIX = VIX vol., MF = Model-free vol., DS = dynamic selection vol.

4.2. Hedging Performances of TAIEX Options

Table 2 shows the out-of-sample hedging performances across the samples and subsamples in accordance with the call option, put option and moneyness. The results indicate that in terms of composite volatility models, the geometric average volatility model had the best hedging performance in terms of the call option, the put option and options with different moneyness. However, there was no significant difference among these three volatility models. In the subsamples, the model-free volatility model had a smaller hedging error regardless of the call option, put option and different moneyness. For different moneyness, the composite volatility model had a better hedging performance across the samples and subsamples; the model-free model yielded a better performance in the subsamples of ATM and OTM call options.

		Whole sample		Out-of-the-money		At-the-	money	In-the-money	
		MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE
	BK EW	4.02(7)	7.25 (7)	3.42 (7)	6.39 (7)	4.44 (6)	7.59(7)	5.26(7)	8.73 (8)
	BK Vega	4.18 (11)	7.85 (11)	3.73 (11)	7.16 (11)	4.99 (11)	8.33 (11)	5.49 (11)	9.21 (11)
	BKELS	4.01 (5)	7.20 (5)	3.41 (6)	6.32 (6)	4.41 (5)	7.52 (5)	5.25 (5)	8.69 (5)
	BKVW	4.22 (9)	7.82 (10)	3.60 (9)	7.02 (10)	4.67 (9)	8.25 (10)	5.40 (9)	9.13 (10)
	BKMW	4.31 (10)	7.35 (8)	3.74 (10)	6.49 (8)	4.97 (10)	7.99 (9)	5.44 (10)	8.71 (6)
Whole	BKCEW	3.90(3)	7.09 (2)	3.27 (3)	6.19 (2)	4.31 (3)	7.43 (3)	5.18 (2)	8.60 (2)
sample	BKCMW	3.90 (4)	7.09 (3)	3.27 (4)	6.19 (3)	4.31 (4)	7.44 (4)	5.18 (3)	8.60 (3)
	BKCGW	3.90(1)	7.08(1)	3.27 (2)	6.17(1)	4.30 (2)	7.43 (1)	5.18(1)	8.59 (1)
	BKVIX	4.13 (8)	7.45 (9)	3.49 (8)	6.55 (9)	4.58 (8)	7.83 (8)	5.34 (8)	8.84 (9)
	BKMF	3.90 (2)	7.10 (4)	3.27 (1)	6.20 (4)	4.30(1)	7.43 (2)	5.18 (4)	8.62 (4)
	BKDS	4.02 (6)	7.24 (6)	3.37 (5)	6.29 (5)	4.46 (7)	7.56 (6)	5.25 (6)	8.71 (7)
	BK EW	4.21 (6)	7.13 (6)	3.81 (6)	6.80 (6)	4.34 (6)	7.08 (6)	4.77 (6)	7.53 (6)
	BK Vega	4.56 (11)	7.71 (11)	4.15 (11)	7.87 (11)	4.68 (11)	7.72 (11)	5.03 (11)	7.83 (11)
	BKELS	4.20(5)	7.10(5)	3.82(7)	6.80(7)	4.31 (5)	7.04 (5)	4.75(5)	7.51 (5)
	BKVW	4.42 (9)	7.74 (10)	4.00 (9)	7.51 (10)	4.55 (9)	7.71 (10)	4.90 (9)	7.84 (10)
	BK_MW	4.49 (10)	7.30 (8)	4.08 (10)	6.86 (8)	4.91 (10)	7.67 (9)	5.03 (10)	7.78 (9)
Call	BK_CEW	4.09 (3)	6.97 (3)	3.65 (3)	6.59 (3)	4.22 (3)	6.95 (3)	4.71 (3)	7.44 (2)
	BK_CMW	4.09 (4)	6.98 (4)	3.65 (4)	6.60 (4)	4.22 (4)	6.95 (4)	4.71 (4)	7.45 (3)
	BK_CGW	4.08 (2)	6.98 (2)	3.64 (2)	6.58 (2)	4.21 (2)	6.94 (2)	4.71 (1)	7.44 (1)
	BK_VIX	4.34 (8)	7.42 (9)	3.86 (8)	6.99 (9)	4.49 (8)	7.42 (8)	4.90 (8)	7.75 (8)
	BK_MF	4.07 (1)	6.96(1)	3.63 (1)	6.57 (1)	4.20(1)	6.92 (1)	4.71 (2)	7.47 (4)
	BK_DS	4.21 (7)	7.19 (7)	3.73 (5)	6.70 (5)	4.37 (7)	7.16 (7)	4.80(7)	7.60(7)
	BK_EW	4.32 (7)	7.37 (7)	3.05 (7)	5.96 (7)	4.54 (6)	8.07 (7)	5.87 (7)	10.02 (8)
	BK_Vega	4.63 (11)	7.86 (11)	3.23 (11)	6.61 (11)	4.86 (11)	8.53 (11)	6.15 (11)	10.68 (11)
	BK_ELS	4.31 (5)	7.30 (6)	3.03 (15)	5.81 (4)	4.51 (6)	7.99 (6)	5.86 (6)	9.97 (7)
	BK_VW	4.51 (9)	7.91 (10)	3.21 (9)	6.51 (10)	4.80 (9)	8.76 (10)	6.01 (10)	10.52 (10)
	BK_MW	4.62 (10)	7.39 (8)	3.41 (10)	6.11 (9)	4.84 (10)	8.31 (9)	5.96 (9)	9.74 (1)
Put	BK_CEW	4.211 (2)	7.20 (2)	2.91 (2)	5.76 (2)	4.41 (3)	7.90 (2)	5.76 (2)	9.84 (3)
	BK_CMW	4.211 (3)	7.21 (3)	2.91 (3)	5.77 (3)	4.41 (4)	7.91 (3)	5.76 (3)	9.84 (4)
	BK_CGW	4.20(1)	7.19(1)	2.91 (1)	5.75 (1)	4.40 (1)	7.89 (1)	5.76 (1)	9.83 (2)
	BK_VIX	4.42 (8)	7.49 (9)	3.14 (8)	6.09 (8)	4.68 (8)	8.24 (8)	5.89 (8)	10.03 (9)
	BK_MF	4.21 (4)	7.23 (4)	2.92 (4)	5.83 (5)	4.40 (2)	7.91 (4)	5.77 (4)	9.87 (5)
Notos E	BK_DS	4.31 (6)	7.30 (5)	3.03 (6)	5.87 (6)	4.56 (7)	7.95 (5)	5.82(5)	9.92 (8)

Table 2: Out-of-the-sample Hedging Performance

Note: EW= equal weighted vol., Vega = Vega weighted vol., ELS = elasticity weighted vol., VW = volume weighted vol., MW = min mean squared errors vol., CEW = composite simple weighted vol., CMW = composite min error-weighted average vol., CGW = composite geometric weighted vol., VIX = VIX vol., MF = Model-free vol., DS = dynamic selection vol.

Overall, the model-free model performed best in terms of ATM and OTM call options; the composite volatility model performed better with respect to ITM call options and ITM, ATM and OTM put options. On the whole, the composite volatility models were the best, followed by the model-free volatility model and the dynamic selection model.

4.3. Error Analysis

Tables 3 and 4 show the results of the regression analysis for out-of-sample option pricing and hedging performances. $\beta 1$, $\beta 2$, $\beta 3$, $\beta 4$, $\beta 5$ and $\beta 6$ represent moneyness, time-to-maturity, risk-free rate, trading volume, open interest and estimates for the call-put dummy variable, respectively. Table 3 indicates that moneyness and time-to-maturity have a significant positive correlation to the absolute value of the pricing error, whereas the trading volume, open interest and call option have a significant negative correlation to the absolute value of the pricing error.

	β ₀	β1	β_2	β3	β_4	β ₅	β_6	$adj-R^2$
DK EW	-38.24500 ^a	45.83000 ^a	0.26111 ^a	0.00001	-0.00018 ^a	-0.00011 ^a	-0.37380 ^a	0.14833
BK_EW	(-38.5030)	(49.6330)	(44.6180)	(1.1102)	(-33.8990)	(-35.5830)	(-3.8216)	
DV Vara	-29.89200 ^a	37.27400 ^a	0.20560 ^a	0.00001	-0.00013 ^a	-0.00011 ^a	-0.10789	0.12050
BK_Vega	(-31.4240)	(41.4160)	(41.1800)	(1.0912)	(-28.2700)	(-37.5440)	(-1.1905)	0.12030
DV EIS	-48.88100 ^a	57.67000 ^a	0.29034 ^a	0.00003	-0.00022 ^a	-0.00013 ^a	-0.11237	0.14844
BK_ELS	(-46.8400)	(60.0990)	(43.8660)	(1.2231)	(-33.0310)	(-35.8670)	(-1.0283)	0.14844
DV VW	-30.97100 ^a	35.82300 ^a	0.29895 ^a	0.00001	-0.00020 ^a	-0.00030 ^a	-0.52583 ^a	0 16269
BK_VW	(-28.8790)	(36.0750)	(46.3300)	(1.1099)	(-15.1370)	(-34.4720)	(-4.9751)	0.16268
DV MW	-29.76600 ^a	39.01500 ^a	0.16903 ^a	0.00002	-0.00014 ^a	-0.00012 ^a	0.90927 ^a	0.09323
BK_MW	(-32.2670)	(44.0470)	(37.9800)	(1.1983)	(-23.7840)	(-36.3970)	(9.8758)	
DK CEW	-33.02700 ^a	40.24100 ^a	0.22355 ^a	0.00001	-0.00014 ^a	-0.00011 ^a	-0.20962 ^a	0.13244
BK_CEW	(-34.5790)	(44.7000)	(43.5200)	(1.1101)	(-30.2440)	(-38.3580)	(-2.2586)	
DV CMW	-32.82100 ^a	40.00300 ^a	0.22305 ^a	0.00001	-0.00014 ^a	-0.00011 ^a	-0.22241 ^a	0.13215
BK_CMW	(-34.3680)	(44.4310)	(43.5050)	(1.0999)	(-30.0740)	(-38.3570)	(-2.3988)	
DK CCW	-32.79200 ^a	39.98000 ^a	0.22205 ^a	0.00001	-0.00014 ^a	-0.00011 ^a	-0.23931 ^a	0.13177
BK_CGW	(-34.4150)	(44.4760)	(43.5520)	(1.1123)	(-30.0260)	(-38.4510)	(-2.5837)	
DV VIV	-32.72600 ^a	41.05200 ^a	0.27762 ^a	0.00003	-0.00015 ^a	-0.00013 ^a	-0.85227 ^a	0.12200
BK_VIX	(-29.5390)	(40.3070)	(38.9160)	(1.2302)	(-26.0360)	(-37.6230)	(-7.3559)	
DV ME	-26.335 ^a	35.99300 ^a	0.28251 ^a	0.00002	-0.00018 ^a	-0.00017 ^a	-0.96200 ^a	0.15754
BK_MF	(-26.1003)	(37.0590)	(43.0890)	(1.1999)	(-26.2160)	(-33.0290)	(-9.9250)	
BK DS	-36.74900 ^a	42.77500 ^a	0.30625 ^a	0.00001	-0.00010 ^a	-0.00011 ^a	0.33420 ^a	0 1 4 7 2 0
—	(-33.3090)	(42.1090)	(44.5080)	(1.0988)	(-20.5930)	(-33.4920)	(3.1650)	0.14729

 Table 3:
 Regression Analysis for Pricing Errors

Note: $AE = |V_M - V_{BL}|$, F/K = moneyness, τ = time-to-maturity, r = risk-free rate, TV = trading volume, OI = open interest, CP = dummy variable for call and put.^a denotes significance at the 5% level.

However, in the minimum error-weighted average volatility and the dynamic selection volatility models, call options have a significant positive correlation to the absolute value of the pricing error. Furthermore, the correlation between the risk-free rate and the absolute value of pricing errors is insignificant. The results above indicate that: (1) ITM tends to result in pricing errors in the volatility models, which accords with Table 1. (2) The pricing error also increases as the time-to-maturity becomes longer. (3) The risk-free rate does not affect the pricing error. (4) Contracts with higher trading volume and open interest have smaller pricing errors; and (5) Call options have smaller pricing errors compared to put options. However, for the minimum error-weighted average volatility and the dynamic selection volatility models, pricing errors for put options are greater than those for call options.

 Table 4:
 Regression Analysis for Hedging Errors

	β ₀	β ₁	β ₂	β3	β4	β ₅	β ₆	adj-R ²
BK_EW	61.16800	0.90373	1.12480	0.00012	0.00399 ^a	-0.00106 ^a	26.40500 ^a	0.00027
	(0.5464)	(0.8541)	(1.9157)	(0.0534)	(5.9738)	(-3.0329)	(2.2246)	0.00027
BK Vega	106.62000	1.42880	0.99444	0.00011	0.00404^{a}	-0.00113 ^a	21.66200	0.00028
DK_vega	(0.9719)	(1.3748)	(1.7479)	(0.0476)	(6.1926)	(-3.3290)	(1.8717)	
BK ELS	-17.20200	0.17899	1.57480 ^a	0.00013	0.00444^{a}	-0.00095 ^a	30.37300 ^a	0.00037
DK_ELS	(-0.1527)	(0.1680)	(2.6790)	(0.0679)	(6.7386)	(-2.7652)	(2.5271)	0.00037
BK VW	194.08000	2.23780 ^a	0.38714	0.00011	0.00399 ^a	-0.00137 ^a	19.46000	0.00021
DK_VW	(1.6181)	(1.9854)	(0.6002)	(0.0501)	(5.8887)	(-3.7798)	(1.5013)	
BK MW	71.23500	1.34560	1.47940 ^a	0.00012	0.00468^{a}	-0.00109 ^a	19.54000	0.00038
BK_WW	(0.6503)	(1.2904)	(2.6046)	(0.0552)	(6.6124)	(-2.9335)	(1.6175)	
BK_CE	81.59300	1.20610	1.11850 ^a	0.00011	0.00424 ^a	-0.00112 ^a	23.50000 ^a	0.00031
W	(0.7414)	(1.1567)	(1.9668)	(0.0482)	(6.5075)	(-3.2910)	(2.0180)	0.00031

 $(AE = \beta_0 + \beta_1 F/K + \beta_2 \tau + \beta_3 r + \beta_4 TV + \beta_5 OI + \beta_6 CP + e)$

BK_CM	83.33300	1.22430	1.11010	0.00011	0.00424^{a}	-0.00112 ^a	23.38700 ^a	0.00031
W	(0.7571)	(1.1739)	(1.9517)	(0.0441)	(6.5043)	(-3.3002)	(2.0078)	0.00031
BK_CG	82.07000	1.21730	1.11830 ^a	0.00011	0.00425 ^a	-0.00112 ^a	23.46100 ^a	0.00031
W	(0.7465)	(1.1685)	(1.9706)	(0.0500)	(6.5311)	(-3.3029)	(2.0165)	0.00031
BK VIX	223.78000	2.58760^{a}	0.83334	0.00010	0.00334 ^a	-0.00132 ^a	21.08200	0.0002
DK_VIA	(1.9151)	(2.3430)	(1.6534)	(0.0387)	(4.8876)	(-3.6694)	(1.6705)	0.0002
DV ME	-10.036	2.22110	4.09190 ^a	0.00012	0.00775^{a}	-0.00136 ^a	17.97797	0.00123
BK_MF	(-0.0587)	(1.3851)	(4.7395)	(0.0554)	(9.7454)	(-3.1062)	(1.4558)	0.00125
BK_DS	186.78	2.44040^{a}	0.73626	0.00011	0.00470^{a}	-0.00144 ^a	16.89000	0.00035
	(1.6019)	(2.2181)	(1.2182)	(0.0467)	(7.0900)	(-4.1336)	(1.3761)	0.00033

 Table 4:
 Regression Analysis for Hedging Errors - continued

Note: $AE = |V_M - V_{BL}|$, F/K = moneyness, τ = time-to-maturity, r = risk-free rate, TV = trading volume, OI = open interest, CP = dummy variable for call and put.^a denotes significance at the 5% level.

Table 4 shows that: (1) the risk-free rate cannot explain the hedging performance. (2) The moneyness and the time-to-maturity have a significant positive correlation to the absolute value of hedging errors. However, this result cannot be proved to be true in general because the moneyness and the time-to-maturity do not have a significant correlation to the absolute value of hedging errors under equal-weighted averaging, Vega-weighted average and minimum error-weighted average volatility models. (3) The trading volume has a significant positive correlation to the absolute value of the hedging errors. (4) The open interest has a significant negative correlation to the absolute value of the hedging errors. (5) The call option is significantly correlated to the absolute value of the hedging errors in five models. Overall, the results above indicate that call options, ITM, longer time-to-maturity, larger trading volume and smaller open interest result in a worse hedging performance.

5. Conclusions

Using the TAIEX options, this study compares the option pricing and hedging performances of different volatility models applied to the Black model. The main characteristic of this study is the incorporation of composite volatility models, including: the equal-weighted average model, the minimum error-weighted average model and the geometric average volatility model. This study also applies dynamic selection model that selects the best volatility model each day to carry out forecasting for the next day. On the whole, in regard to pricing, the model-free model has the best performance, and the composite volatility models come in second. The dynamic selection model does not have outstanding performance. In regard to hedging, the composite volatility models have the best performance, followed by the model-free model and the dynamic selection model has the worst performance. The Black option model is used widely in practice, and the estimated volatility affects pricing and hedging performances. The design of this research is to make a contribution to the academic field as well as a reference for the industry.

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