International Research Journal of Finance and Economics
ISSN 1450-2887 Issue 99 October, 2012

http://www.internationalresearchjournaloffinanceandeconomics.com

A Note on the Edgeworth Box

Alamedin Bannaga
(PhD Manchester University) Address: The Arab Monetary Fund – Abu Dhabi
P.O. Box: 2818, United Arab Emirates
E-mail: abannaga@hotmail.com
Tel: 0097126215000

Abstract

The basic idea of this paper is to provide a different specification for the relationship between the two consumers trading two commodities in the Edgeworth box. We use this relationship to generate terms of exchange for each consumer. A different interpretation for the rule of exchange emerges out.

Keywords: Edgeworth box, Exchange, Trade, Pareto Efficiency
JEL Classification Codes: D11, D51, D61

1. Introduction

Edgeworth box is a popular tool in general equilibrium analysis. It was developed by Francis Ysidro Edgeworth (1881) and later by Arthur Bowley (1924). The box allows the study of exchange and trade of commodities between two individuals using the concept of indifference curves. In this paper, we build our analysis on a circle inscribed inside a box.

The rest of the paper is organized as follows: in the next section we offer a brief description for the theory of the Edgeworth box. In the third section we provide some assumptions followed by discussion in the fourth section. The fifth section is a conclusion.

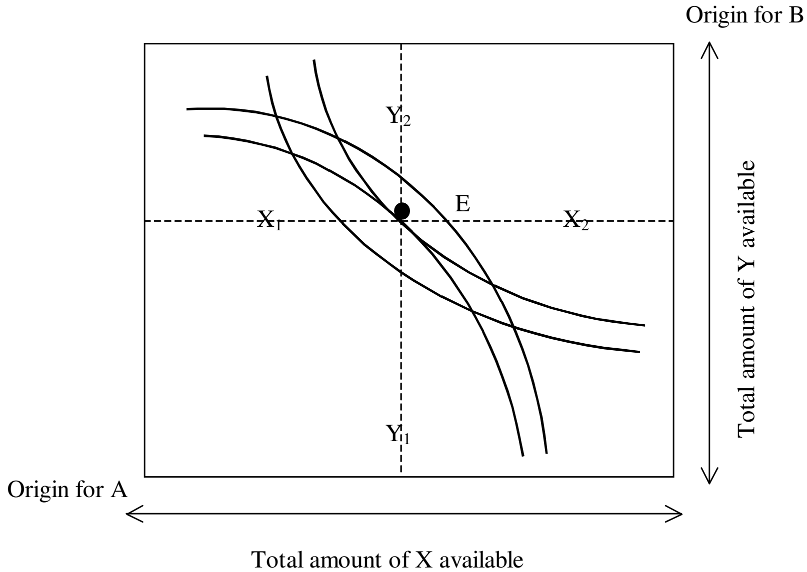
2. Edgeworth box Theory

The Edgeworth Box is diagram showing all possible allocations of either two goods between two people or of two inputs between two production processes. If we have two individuals A and B and two commodities X and Y each individual has an initial endowment of commodities X and Y before trade. The figure (1) shows the Edgeworth Box for the two consumers. The height of the Edgeworth box represents the total amount of commodity (Y) available and the width of the Edgeworth box represents the total amount of commodity (X) available. The Edgeworth box suggests that moving in the northeast direction will make the first person (A) better off and moving to the southwest direct will make the second person (B) better off.

In the Edgeworth diagram, wherever one of the indifference curves of the first consumer (A) touches an indifference curve of the second consumer (B), a unique combination of the two goods is identified that yields both consumers a maximum value. The slope of the indifference curve is (dy/dx = MUx / MUy = MRSxy) and the Pareto optimality may be properly defined as that point where: $MRS^A = MRS^B$. The relative prices between the two commodities represents the terms of trade between the two individuals. A competitive equilibrium will be established at that point (a Pareto Optimum), where:

$$MRS^A_{xy} = MRS^B_{xy} = P_x / P_y$$

Figure 1: The Edgeworth Box

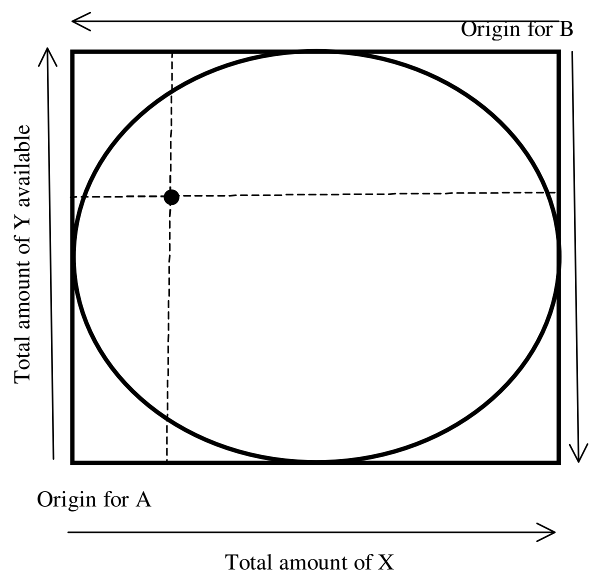


The figure (1) shows the total amount of the first commodity X is (X_1 + X_2) and the total amount of the second commodity is (Y_1 + Y_2). The point (E) is the tangency point between the first and second consumer indifference curves. It is an equilibrium point. At this point the first consumer has the amount X_1 of the first commodity and the amount Y_1 of the second commodity. The second consumer has X_2 of the first commodity and Y_2 of the second commodity. The point (E) is a Pareto Optimal point. We will focus on the point (E) on our following discussion.

3. Assumptions

We make a simple assumption that the Edgeworth box is a square and we have a circle inscribed inside the square as shown in figure (2). The exchange and trade between the two consumers can take place anywhere inside the square.

Figure 2: A circle inside the Edgeworth Box



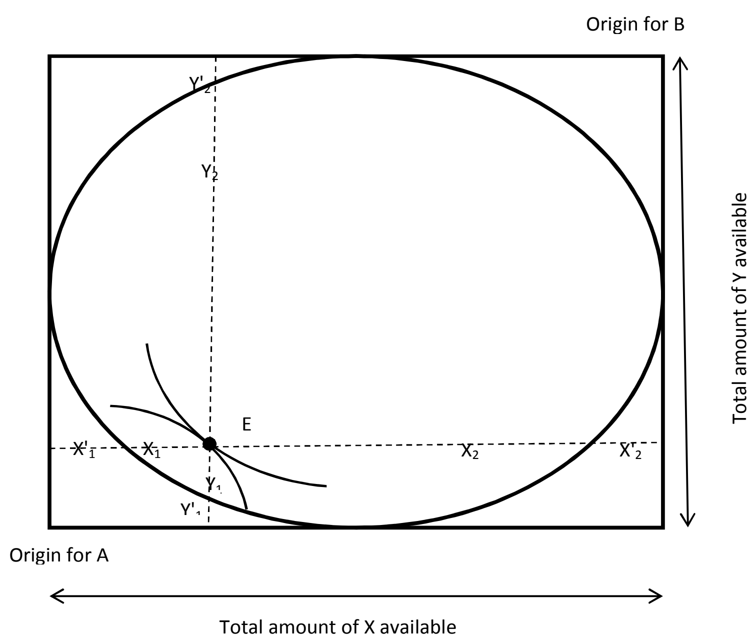
Cornwall (1984: PP:6) described the Edgeworth box by saying that: “To read the coordinates according to the second consumer axis, you should either stand on your head or turn the page 180^0 (he added: Yoga is useful)”. If there is a need to rotate or turn the page 180^0 to understand the Edgeworth box, it indicates that for each Edgeworth box, there exists a circle that can help explaining the exchange process.

Moreover, using a circle to explain exchange and trade has its attractiveness since the general equilibrium theory is based on the Brouwer fixed-point theorem. The simplest form of this theorem is a continuous functions *f* from a disk *D* to itself.

4. Discussion

We will use the figure (3) for our discussion. It shows a circle inside the Edgeworth Box. The point (E) is the equilibrium point. At this point, the first consumer has the amount of ($X_1 + X'_1$) of commodity X. Where X_1 is that proportion of X consumed by the first consumer and lies inside the circle. On the other hand, X'_1 is that the proportion of X consumed by the first consumer and lies outside the circle. Similarly, the total amount of Y consumed by the first consumer is ($Y_1 + Y'_1$). The proportion inside the circle is Y_1 and the proportion outside the circle is Y'_1. The second consumer consumes commodities X and Y. With proportion (X_2 , Y_2) inside the circle and (X'_2 , Y'_2) outside the circle.

Figure 3: A circle inside the Edgeworth Box



Now we focus on the point (E) as a point inside the circle. It is a point of intersection of two chords inside a circle. Then we can apply the power of a point theorem (by Jacob Steiner, 1826) to the point (E). The theorem says:

“If two chords intersect in a circle, the product of the lengths of the segments of one chord equals the product of the segments of the other”. The two chords need not be perpendicular for the theorem to apply.

Applying the theorem to the circle in figure (3), we get:

$$\boldsymbol{X_1 X_2 = Y_1 Y_2} \tag{1}$$

This equation provides a rule of exchange inside the circle.

From this equation:

$$\boldsymbol{X_1 / Y_1 = Y_2 / X_2} \tag{1a}$$

This equation allows us to shift the focus from the absolute amount of the commodities ($X_1 + X_2$; $Y_1 + Y_2$) or total endowment of the commodity, as normally used in the economic literature to the ratios of these commodities consumed by each customer.

Outside the circle, the four areas between the square and the circle are congruent. This means:

$X'_1 = X'_2$ and $Y'_1 = Y'_2$

This implies that exchange is meaningless and we can drop these areas as irrelevant for efficient exchange. Efficient exchange can be restricted to the area inside the circle only.

Now put:

$$X_1 + X_2 = X_c \quad (2)$$

This is the proportion of commodity X inside the circle.

Let $X_1 = \alpha X_c$ and $X_2 = (1 - \alpha) X_c$ (3)

α: Represents the share of the first consumer in X_c and $(1 - \alpha)$ represents the share of the second consumer in X_c. Similarly,

$$Y_1 + Y_2 = Y_c \quad (4)$$

Let $Y_1 = \beta Y_c$ and $Y_2 = (1 - \beta) Y_c$ (5)

β: Represents the share of the first consumer in Y_c and $(1 - \beta)$ represents the share of the second consumer in Y_c.

Substitute equations (3) and (5) in equation (1):

$$X_1 X_2 = Y_1 Y_2 \quad (1)$$

$$\alpha X_c . (1 - \alpha) X_c = \beta Y_c . (1 - \beta) Y_c$$

Giving: $\alpha (1 - \alpha) X^2_c = \beta (1 - \beta) Y^2_c$ (6)

Then, totally differentiate equation (6)

$$2\alpha (1 - \alpha) X_c dX_c = 2\beta (1 - \beta) Y_c dY_c$$

Giving: $dY_c / dX_c = \alpha (1 - \alpha) X_c / \beta (1 - \beta) Y_c$ (7)

But $dY_c / dX_c = MRS_{YX}$

Hence, $MRS_{YX} = \alpha (1 - \alpha) X_c / \beta (1 - \beta) Y_c$ (8)

We can see that from equation (3) and equation (5)

$X_1 = \alpha X_c$ and $Y_1 = \beta Y_c$

When we substitute these values in equation (8) we get:

$$MRS_{YX} = (1 - \alpha) X_1 / (1 - \beta) Y_1 \quad (9)$$

Similarly,

$$MRS_{YX} = \alpha X_2 / \beta Y_2 \quad (10)$$

Equation (9) indicates that the MRS for the first consumer depends on the values of (α , β) these are the proportion of total amount of commodities X and Y owned by this consumer. Similarly, equation (10) relates the MRS of the second consumer to the proportion he owned of each commodity.

However, $MRS_{YX} = P_1 / P_2$

This means that:

$$(1 - \alpha) X_1 / (1 - \beta) Y_1 = P_1 / P_2 \quad (11)$$

And

$$\alpha X_2 / \beta Y_2 = P_1 / P_2 \quad (12)$$

Equations (11) and (12) describe the terms of exchange of commodities for each consumer. It is clear that the exchange depends on the values of (α, β, P_1, P_2). These values are related to the shares of the consumer in the total amount of the commodity and the price of each commodity.

5. Conclusions

The paper tries to explain the exchange taking place between two consumers in the Edgeworth Box. In practice, we normally rotate the page 180^0 to be able to read the second consumer axis. This means that we can make use of a circle geometry to explain the exchange process.

Assuming that the equilibrium point lies in a circle inscribed inside a square allows us to apply the power of a point theorem and produce a new equation (equation 1) giving a new rule of exchange between the two consumers. This equation is independent of the underlying utility function assumptions. Further, the equation allowed us to suggest that the terms of exchange between two consumers depends mainly on the consumer's share of the total amount of the commodity in question.

References

[1] Bowley, A. L.,(1924) *The Mathematical Groundwork of Economics: An Introductory Treatise.* Clarendon Press, Oxford.

[2] Cornwall, R. C.,(1984) *Introduction to the Use of General Equilibrium Analysis*. North Holland, Elsevier Science Publishers B.V, The Netherlands.

[3] Edgeworth, F. Y., (1881) *Mathematical Physics*. Paul Kegan, London.

[4] Estrin, S., Laidler, D., Dietrich M., (2008) *Microeconomics*. Fifth Edition, Pearson Education Ltd.

[5] Varian, H. R., (2003) *Intermediate Microeconomics: A Modern Approach*. Sixth Edition, W. W. Norton Company.

[6] Wikipedia, Free Encyclopedia Web.: http://en.wikipedia.org/wiki/Regular_polygon