Leverages and Stock Return Volatility under Recessions

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Abstract

This paper studies the fundamental driving forces of the high market-level stock return volatility under recessions. Based on a production-based asset pricing model, important roles of both financial and operating leverages and their simultaneous amplification effect on the volatility are implied. Quantitative results show that both financial and operating leverages significantly explain the stock return volatility, and this explanatory power becomes distinctive during recessions. Operating leverage in particular is the crucial factor that derives a higher volatility during recessions, predicting a higher stock return over time. This relation also implies a source of value premium at a firm level.

Keywords: Asset Pricing, Leverage, Volatility, Production Economy **JEL Classification Codes:** G01, G12

1. Introduction

Stock return volatility was unusually high during the Great Depression of 1929 – 1939 (Officer (1973)). Schwert (1989) shows that stock return volatility was higher during recessions and around the major banking panics of the nineteenth and early twentieth centuries. A large body of literature has attempted to explain the movements in aggregate stock market volatility, but they seem unsatisfactory, especially when attempting to explain such movements in bad economic times (See Black (1976), Christie (1982), Schwert (1990), Duffee (1995), and Aydemir, Gallmeyer, and Hollifield (2006) among others). This naturally raises the question of why the stock return is so volatile, and so particularly when the aggregate economy undergoes hardship.

A well-known stylized fact from previous literature is that stock returns are negatively correlated to the subsequent return volatility, and vice versa. This phenomenon is asymmetrically more pronounced when current low stock returns are associated with increased future volatility. This asymmetric volatility has been explained by two observationally equivalent hypotheses: first, Black (1976) and Christie (1982) argue that when the value of stocks drops relative to the value of bonds, firms become more financially leveraged. This high financial leverage, in turn, causes an increase in the future volatility of the return on stocks. Empirical evidence generally confirms that at the aggregate market level, stock return volatility is asymmetrically related to negative and positive returns. But this so-called financial leverage effect alone is insufficient to explain the observed negative correlation,

particularly when the volatility is very high, such as in the Great Depression (Schwert (1989)). Second, Poterba and Summer (1986) point out that an unexpected increase in volatility raises the expected future volatility, thus resulting in an immediate negative impact on the current stock price. This, then, implies negative correlation between expected volatility and current returns on stock. This other negative relation is called the volatility feedback effect. Many studies, including French et al. (1987) and Campbell and Hentschel (1992), find evidence in support of this volatility feedback effect, but the effect alone is not enough to account for the negative relation, as with the financial leverage effect. Bekaert and Wu (2000) study the financial leverage and volatility feedback effects simultaneously and disentangle both effects. In addition to these two explanations, behavioral economists argue that the stylized fact could be due to noise trading from irrational investors. See Chen, Hong, and Stein (2001), Goyal and Santa Clara (2003), and Hong and Stein (2003) among others.

We approach the volatility feedback effect newly and systematically with particular emphasis on instances of the phenomenon that occur during bad economic times. We argue that the countercyclical operating leverage is the source of the strong volatility feedback effect. When the economy is expected to be bad, firms reduce their investments because they need to decrease their production relative to lower expected demands. In particular, since fixed assets such as factories, office buildings, and machines are less liquid, firms are more likely to reduce their variable assets, such as investments in R&D or firm-specific human capital. This unbalanced decrease in asset structure causes firms' high operating leverage. If firms happen to have large fixed assets, operating profits or losses will fluctuate extensively, which in turn results in high expected stock return volatility. This anticipation of increased volatility leads to a decline in stock price at the same time, which is, by definition, the volatility feedback. Therefore, both financial and operating leverages predict a higher stock return volatility, particularly under recessions. We combine these two empirically identical but theoretically different effects into one parsimonious, production-based asset pricing model to explain market-level high stock return volatility under recession conditions.

The model implies that the volatility effect arising from increasing levels of operating leverage causes immediate and severe stock price drops relative to bond price, which in turn increases firms' financial leverage. Hence, the increasing stock return volatility caused by increasing levels of operating leverage implies amplification by the simultaneous increase in the level of financial leverage. Even though the amplification effect is silent, the quantitative results of the model demonstrate that each leverage is the main driving forces of recession-period high stock return volatility. In particular, we found that operating leverage played a stronger role in creating volatility, which is linked to significant positive relation between operating leverage and stock return as in Novy-Marx (2007), Gulen, Xing, and Zhang (2008), and Garcia-Feijoo and Jorgensen (2010). The risk stemming from these financial and operating leverages predicts a higher expected return over time, since investors should be compensated. We confirm that both leverages are positively associated with stock returns. In addition, asymmetric fixed and variable capital adjustment costs are implied based on associated parameters in the model, because liquidating fixed assets is a less flexible option. Moreover, we estimate timevarying parameters related to the fixed capital adjustment costs. A higher adjustment cost is implied when the economy is in a bad time because divesting fixed capital becomes more difficult. This is naturally connected to a higher operating leverage under recessions because firms are more likely to reduce their variable capital sources. The data clearly show that the level of operating leverage is higher during recession periods.

The contribution of this paper is fourfold. First, to our knowledge, this paper is the first to incorporate financial leverage effect and operating leverage effect under a production-based asset pricing model. Jermann (1998) briefly argues that financial leverage might improve the models by providing a source of high volatility of stock return. Second, our study is particularly focused on recession periods. Higher financial and operating leverages are expected, and this implies a higher stock return volatility. Third, as Schwert (1989) argues, regression-based empirical analyses do not test for causes of stock return volatility. Our structural estimation approach, which uses GMM, relieves this critical issue, and the model provides determinants of stock return and its volatility based on

fundamental factors. Finally, the model provides a possible source of the value premium and the pricing factor at firm levels, similar to the studies of Zhang (2005) and Cooper (2006).

The paper is organized as follows: section 2 presents the production-based asset pricing model. Section 3 discusses the econometric methodology and data. Section 4 reports the empirical results. Section 5 concludes the argument.

2. Model

The main frameworks in the economy are similar to those in Liu, Whited, and Zhang (2009) and Li and Liu (2010). Among the investment-based *Q*-theory models, our model especially relies on Li and Liu (2010), since they also consider two different capital assets by usefully augmenting the model proposed in Liu, Whited, and Zhang (2009). However, we apply the model to an aggregate market level in a more parsimonious manner. We use the capital accumulation equation in Cochrane (1991) since he also studies the market-level stock return behavior based on his production asset pricing model. More importantly, we focus on the financial and operating leverage effects on market level stock returns, particularly under recessions, while their interests are in explaining cross-sectional stock returns. In addition, we primarily study the financial and operating leverages, which are endogenous variables from the data.

Firms

In a discrete and infinite time horizon, every firm maximizes the present value of an infinite stream of the firm's free cash flow at each time period by optimally determining how much labor to hire, how many bonds to issue, and which fixed and variable capitals to invest. The output or revenue of firm *j* at time *t*, y_{j} , is produced by following Cobb-Douglas production technology:

$$Y_{jt} = \left[\left(k_{jt}^F \right)^y \left(k_{jt}^V \right)^{1-y} \right]^{\alpha} \left[A_{jt} l_{jt} \right]^{1-\alpha}$$

$$\tag{1}$$

where k_{jt}^F and k_{jt}^V are the levels of fixed and variable assets, respectively, and A_{jt} denotes the firmspecific exogenous productivity shock. l_{jt} is the human capital from the factor market. α represents the capital share of total output due to the nature of the output function, and γ is the elasticity of substitution between fixed and variable assets. Following previous literatures, we impose α equal to 0.36 (Santoro and Wei (2010)). As in Cochrane (2001), both the firm's fixed and variable capital stocks follow the intertemporal accumulation equation with adjustment costs as follows:

$$k_{jt+1}^{x} = \left(1 - \delta_{jt}^{x}\right) \left[k_{jt}^{x} + i_{jt}^{x} - \Phi_{jt}^{x}\right], x = F, V$$
(2)

where i_{jt}^x is the additional investment of fixed (*F*) and variable (**V**) capitals determined by firm *j* at time *t*, and δ_{jt}^x is the corresponding depreciation rate. Both additional investments in fixed and variable assets involve adjustment costs, Φ_{jt}^x . The adjustment cost functions are defined as

$$\Phi_{jt}^{F}\left(i_{jt}^{F},k_{jt}^{F}\right) \equiv \frac{a}{2} \left(\frac{i_{jt}^{F}}{k_{jt}^{F}}\right)^{\rho} i_{jt}^{V}$$

$$\Phi_{jt}^{V}\left(i_{jt}^{V},k_{jt}^{V}\right) \equiv \frac{b}{2} \left(\frac{i_{jt}^{V}}{k_{jt}^{V}}\right)^{\psi} i_{jt}^{V}$$
(3)

where Φ is increasing and convex in i and is decreasing in k, and displays constant returns to scale in i and k such that $\Phi(i,k) = i \frac{\partial \Phi(i,k)}{\partial i} + k \frac{\partial \Phi(i,k)}{\partial k}$ if ρ and ψ are greater than 1. We set ρ equal to 2 in order to be consistent with Liu, Whited, and Zhang (2009) and Li and Liu (2010), but we allow ψ to be

determined from the model calibration. Notice that the investment adjustment function and the production function follow the characteristic of the constant return to scale.

Firms produce outputs by using both equity and debt financing. As in Hennessy and Whited (2007) and Liu, Whited, and Zhang (2009), firms issue one-period debt, which has the outstanding value of b_{jt} at the beginning of period t with the required gross return of r_{jt}^b . In addition, firms continuously issue the new debt b_{jt+1} at the end of period t. Then, the free cash flow to the equity holders of firm j at period t, d_{jt}^E , is

$$d_{jt}^{E} = \left(1 - \tau_{jt}^{c}\right) \left(y_{jt} - \overline{\omega}_{t}l_{jt} - \Phi_{jt}^{F} - \Phi_{jt}^{V}\right) - i_{jt}^{F} - i_{jt}^{V} + \tau_{jt}^{c} \delta_{jt}^{F} \left(k_{jt}^{F} + i_{jt}^{F}\right) - \left[1 + \left(r_{jt}^{b} - 1\right)\left(1 - \tau_{jt}^{c}\right)\right] b_{jt} + b_{jt+1}$$

$$(4)$$

where ϖ_t is the real wage of the human capital and τ_{jt}^c is the corporate income tax rate on firm *j* at time *t*. The terms $\tau_{jt}^c \delta_{jt}^F (k_{jt}^F + i_{jt}^F)$ and $(r_{jt}^b - 1)\tau_{jt}^c b_{jt}$ in $-[1 + (r_{jt}^b - 1)(1 - \tau_{jt}^c)]b_{jt}$ from equation (4) represent that depreciation tax shield and interest tax shield, respectively.

Equilibrium

In the infinite and discrete time horizon, the value of a firm's equity at time *t* can be expressed as

$$p_{jt} = d_{jt}^{E} + E_{t} \left[M_{t+1} p_{jt+1} \right]$$
(5)

where M_{t+1} is the stochastic discount factor from t to t+1 and p_{jt} is the cumulative value of the free cash flow accrued to the equity holders at time t. Then the after-dividend firm value at the end of period t is

$$v_{jt} = p_{jt} - d_{jt}^{E} + b_{jt+1}$$
(6)

Firms maximize the values of v_{jt} at each period by optimally determining the i_{jt}^F , i_{jt}^V , k_{jt+1}^F , k_{jt+1}^V , b_{jt+1} , and l_{jt} under the constraints of equations (2) and (3) above. Solving this maximization problem exhibits the firm's return on equity, r_{jt+1}^E ,

$$r_{jt+1}^{E} = \frac{r_{jt+1}^{A} - w_{jt}r_{jt+1}^{b_after-tax}}{1 - w_{jt}}$$

$$= \frac{\theta_{jt}r_{jt+1}^{F} + (1 - \theta_{jt})r_{jt+1}^{V} - w_{jt}r_{jt+1}^{b_after-tax}}{1 - w_{jt}}$$
(7)

where the return on asset $\binom{r_{jt+1}^A}{p}$ is equivalent to the weighted average of the returns on fixed asset $\binom{r_{jt+1}^F}{p}$ and variable asset $\binom{r_{jt+1}^V}{p}$ and to the weighted average of the return on firm's equity $\binom{r_{jt+1}^E}{p}$ and the after-tax return on its debt $\binom{r_{jt+1}^b}{p}$. Specifically:

$$r_{jt+1}^{A} = \theta_{jt} r_{jt+1}^{F} + (1 - \theta_{jt}) r_{jt+1}^{V} = (1 - w_{jt}) r_{jt+1}^{E} + w_{jt} r_{jt+1}^{b_{-after-tax}}.$$
(8)

In addition, r_{jt+1}^{A} , r_{jt+1}^{r} , and r_{jt+1}^{v} satisfy the following relations:

$$r_{jt+1}^{A} = \frac{\left(1 - \tau_{jt+1}^{c}\right)\left(\frac{\gamma y_{jt+1}}{k_{jt+1}^{F}}\right) - \left(1 - \tau_{jt+1}^{c}\right)\frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}} + \tau_{jt+1}^{c}\delta_{jt+1}^{F} + q_{jt+1}^{F}(1 - \delta_{jt+1}^{F})\left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}}\right)}{\left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}}\right) - \left(1 - \tau_{jt+1}^{c}\right)\frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{F}}\right)}{\left(1 - \sigma_{t}^{F}\right)\left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial t_{jt+1}^{F}}\right)} - \left(1 - \tau_{jt}^{c}\right)\left(\frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{F}}\right)}{\left(1 - \sigma_{t}^{F}\right)\left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial t_{jt+1}^{F}}\right)}\right) + \left(\frac{1 + \left(1 - \tau_{jt}^{c}\right)\left(\frac{\partial \Phi_{jt+1}^{V}}{\partial t_{jt+1}^{F}}\right)}{\left(1 - \sigma_{t}^{F}\right)\left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial t_{jt+1}^{F}}\right)}\right) + \left(\frac{1 + \left(1 - \tau_{jt}^{c}\right)\left(\frac{\partial \Phi_{jt+1}^{V}}{\partial t_{jt+1}^{F}}\right)}{\left(1 - \sigma_{t}^{F}\right)\left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial t_{jt+1}^{F}}\right)} + \tau_{jt+1}^{c}\delta_{jt+1}^{F} + q_{jt+1}^{F}(1 - \delta_{jt+1}^{F})\left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial t_{jt+1}^{F}}\right)}\right)}{\left(1 - \sigma_{t}^{F}\right)\left(1 - \sigma_{t}^{F}\right)\left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial t_{jt+1}^{F}}\right)} - \tau_{jt}^{c}\delta_{t}^{F}}\right)}$$

and

$$r_{jt+1}^{V} = \frac{(1 - \tau_{jt+1}^{c}) \left[(1 - \alpha) \frac{\gamma y_{jt+1}}{k_{jt+1}^{V}} - \left(\frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}} \right) \right] + q_{jt+1}^{V} (1 - \delta_{jt+1}^{V}) \left(1 - \frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}} \right)}{\left[\frac{1 + (1 - \tau_{jt}^{c}) \left(\frac{\partial \Phi_{jt+1}^{V}}{\partial i_{jt+1}^{V}} \right)}{(1 - \delta_{t}^{V}) \left(1 - \frac{\partial \Phi_{jt+1}^{V}}{\partial i_{jt+1}^{V}} \right)} \right]}$$
(9)

 θ_{jt} and w_{jt} denote the ratio of fixed asset value to total asset value and the ratio of debt value to overall firm value at the end of period *t* as follows:

$$\theta_{jt} = \frac{k_{jt}^{*}}{k_{jt}^{F} + k_{jt}^{V}}$$

$$w_{jt} = \frac{b_{jt+1}}{v_{jt}} = \frac{b_{jt+1}}{p_{jt} - d_{jt}^{E} + b_{jt+1}}$$
(10)

The return on equity from period t to t+1 and the after-tax return on debt are

$$r_{jt+1}^{E} = \frac{p_{jt}}{p_{jt} - d_{jt}^{E}}$$

$$r_{jt+1}^{b_after_tax} = r_{jt+1}^{b} - \tau_{jt+1}^{c} (r_{jt+1}^{b} - 1)$$
(11)

The proof, which follows Li and Liu (2010), is provided in appendix A.

As seen in equation (7), the return on equity is a function of the ratio of fixed asset value to total asset value, the ratio of debt value to overall firm value, the return on fixed asset and variable asset, and the after-tax return on the firm's debt. By definition, the ratio of debt value to overall firm value can be considered a firm's financial leverage. Consistent with previous literature, the ratio of fixed asset value to total asset value is proxied for the operating leverage of a given firm (See Mandelker and Rhee (1984) and Garcia-Feijoo and Jorgensen (2010), among others). In order to see the qualitative implications of this financial leverage and operating leverage, we differentiate equation

(7) with respect to W_{jt} and θ_{jt} as follows:

$$\frac{\partial r_{j_{t+1}}^{E}}{\partial w_{j_{t}}} = \frac{\theta_{j_{t}} r_{j_{t+1}}^{F} + (1 - \theta_{j_{t}}) r_{j_{t+1}}^{V} - r_{j_{t+1}}^{b}}{\left(1 - w_{j_{t}}\right)^{2}}$$

$$\frac{\partial r_{jt+1}^E}{\partial \theta_{jt}} = \left(\frac{1}{1 - w_{jt}}\right) \left(r_{jt+1}^F - r_{jt+1}^V\right) \tag{12}$$

The fact that $\partial r_{jt+1}^E / \partial w_{jt}$ is the inverse function of $(1 - w_{jt})^2$ implies that the return on equity is positively related to the level of financial leverage. $\partial r_{ii+1}^E / \partial \theta_{ii}$ in equation (12) shows that the return on equity is also positively related to the operating leverage, as long as the return on fixed assets is greater than that on variable assets. These necessary conditions are confirmed, as the mean of the returns on fixed capital assets turns out to be higher than that for variable capital assets. See Section 4 for details. When both leverages change, the return on equity is more positively related to the simultaneous effect of increases in both leverages, and the magnitude of the simultaneous effect is greater than the mere sum of the two effects when taken separately. This relation implies that equity risks stemming from these two different leverages may greatly amplify each other when they increase simultaneously. In order to examine this possible amplification effect, the volatility of the equity returns is provided. Based on the assumption that the volatility of the after-tax bond returns is zero (for simplicity), $Var[r_{jt+1}^b] = 0$, the volatility of the equity returns can be expressed as

$$\frac{\partial^2 r_{j_{t+1}}^E}{\partial w_{j_t} \partial \theta_{j_t}} = \underbrace{\left(\frac{1}{1 - w_{j_t}}\right)^2 \left(r_{j_{t+1}}^F - r_{j_{t+1}}^V\right)}_{Both \ Effects} > \underbrace{\frac{\theta_{j_t} r_{j_{t+1}}^F + (1 - \theta_{j_t}) r_{j_{t+1}}^V - r_{j_{t+1}}^b}{\left(1 - w_{j_t}\right)^2}}_{Financial \ Leverage \ Effect} + \underbrace{\left(\frac{1}{1 - w_{j_t}}\right) \left(r_{j_{t+1}}^F - r_{j_{t+1}}^V\right)}_{Operating \ Leverage \ Effect}$$
(13)

$$Var[r_{jt+1}^{E}] = Var\left[\frac{\theta_{jt}r_{jt+1}^{F} + (1-\theta_{jt})r_{jt+1}^{V} - w_{jt}r_{jt+1}^{b}}{1-w_{jt}}\right]$$
$$= \left(\frac{\theta_{jt}}{1-w_{jt}}\right)^{2} Var[r_{jt+1}^{F}] + \left(\frac{1-\theta_{jt}}{1-w_{jt}}\right)^{2} Var[r_{jt+1}^{V}] + 2\left(\frac{\theta_{jt}}{1-w_{jt}}\right)\left(\frac{1-\theta_{jt}}{1-w_{jt}}\right) cov[r_{jt+1}^{F}, r_{jt+1}^{V}]$$
(14)

Further assuming that the covariance term in equation (14) is small enough, equation (14)becomes (The variance of returns on bonds and the covariance between returns on fixed and variable capitals are very small numbers in our empirical results.)

$$Var\left[r_{jt+1}^{E}\right] = \left(\frac{\theta_{jt}}{1 - w_{jt}}\right)^{2} Var\left[r_{jt+1}^{F}\right] + \left(\frac{1 - \theta_{jt}}{1 - w_{jt}}\right)^{2} Var\left[r_{jt+1}^{V}\right]$$
(15)

In a similar way to the action of equation (12), the derivatives of the equity return volatility with respect to W_{jt} and θ_{jt} are as follows:

$$\frac{\partial Var\left[r_{jt+1}^{E}\right]}{\partial w_{jt}} = \frac{2}{\left(1 - w_{jt}\right)^{3}} \left[\theta_{jt}^{2} Var\left[r_{jt+1}^{F}\right] + (1 + \theta_{jt})^{2} Var\left[r_{jt+1}^{V}\right]\right]$$

$$\frac{\partial Var\left[r_{jt+1}^{E}\right]}{\partial \theta_{jt}} = \frac{1}{\left(1 - w_{jt}\right)^{2}} \left[2\theta_{jt} Var\left[r_{jt+1}^{F}\right] + (2\theta_{jt} - 2) Var\left[r_{jt+1}^{V}\right]\right]$$
(16)

(10)

As seen in equation (16), the volatility of equity return is an increasing function of the financial leverage w_{jt} . However, the volatility increases in the level of operating leverage unless $Var[r_{jt+1}^F] < Var[r_{jt+1}^V]$ is measurably satisfied.

Market

Up to this point, we have discussed the possible risk factors, the financial leverage, and the operating leverage at the level of the individual firm. If these two risk factors from firm-specific characteristics can be effectively diversified away, the significant relations between these factors and equity return/volatility may not be observed on the level of the firm. However, market-level adverse economic shocks such as recessions, which cannot be immunized at the individual firm level, can result in both higher operating leverages and higher financial leverages, at least, to an average firm or overall market. The effects of aggregate shocks on operating and financial leverages at the individual firm level should be less pronounced due to the other firm specific factors that affect levels of both leverages. Therefore, market-level study is more appropriate. Hence, we particularly examine whether the model predictions are distinctive at the market level. Firms on average must decrease the level of asset capacity in order to deal with future decreases in demand for their output when an economic downturn is expected. Nevertheless, firms are likely to decrease variable capitals instead, due to their flexibility. Downsizing variable assets by decreasing the level of corresponding investments is more flexible compared to the liquidation of fixed assets, especially under recessions. For example, they can decrease their R&D investments, which are not immediately necessary to production, while selling the firm's machines or equipment would prove costly, especially when the economy is bad. The firms' aggregate action increases the market level operating leverage, which causes overall equity price drops due to additional volatility in the average firm's income streams. Relatively higher equity price drop compared to bond prices implies a higher market-level financial leverage. Therefore a higher market-level stock return volatility is expected under recessions.

3. Econometric Methodology and Data

To obtain the estimated parameters in our model, two main tests are conducted for market-level stock return and its volatility based on Liu, Whited, and Zhang (2009). First, we examine whether expected stock returns from the data are equal to expected equity returns from the model in equation (7).

$$E\left[R_{t}^{E} - \frac{\theta_{t}R_{t+1}^{T} + (1 - \theta_{t})R_{t+1}^{IT} - w_{t}R_{t+1}^{B}}{1 - w_{t}}\right] = 0 \ (Market \ Level)$$
(17)

Second, we also examine whether equity return variations from the model are equal to its variations from the data.

$$E\left[\left(R_{t}^{E}-E\left[R_{t}^{E}\right]\right)^{2}-\left(\frac{\theta_{t}R_{t+1}^{T}+(1-\theta_{t})R_{t+1}^{T}-w_{t}R_{t+1}^{B}}{1-w_{t}}-E\left[\frac{\theta_{t}R_{t+1}^{T}+(1-\theta_{t})R_{t+1}^{T}-w_{t}R_{t+1}^{B}}{1-w_{t}}\right]\right)^{2}\right]=0 (Market Level) (18)$$

However, since no parameters exist to precisely satisfy the model prediction shown in equations (17) and (18) at every state and period of the world, we utilize the model errors from the moment conditions to test the model as follows:

$$e^{u} = E_{T} \left[R_{t}^{E} - \frac{\theta_{t} R_{t+1}^{H} + (1 - \theta_{t}) R_{t+1}^{H} - w_{t} R_{t+1}^{B}}{1 - w_{t}} \right] (Market \ Level)$$

$$e^{\sigma^{2}} = E_{T} \left[\left(R_{t}^{E} - E_{T} \left[R_{t}^{E} \right] \right)^{2} - \left(\frac{\theta_{t} R_{t+1}^{T} + (1 - \theta_{t}) R_{t+1}^{H} - w_{t} R_{t+1}^{B}}{1 - w_{t}} - E_{T} \left[\frac{\theta_{t} R_{t+1}^{T} + (1 - \theta_{t}) R_{t+1}^{H} - w_{t} R_{t+1}^{B}}{1 - w_{t}} \right] \right)^{2} \right] (Market \ Level)$$

$$(19)$$

where E_T [] denotes the sample mean of variables in the brackets. e_t^u and $e_t^{\sigma^2}$ represent the expected return and variation errors, respectively. Errors can occur due to either measurement or model specification issues. As Liu, Whited, and Zhang (2009) state, measuring variable assets, finding a correct function of investment adjustment costs, and assuming constant return to scale of the output function could explain these issues. The assumption that both e^u and e^{σ^2} have zeros on average allows us to utilize two moment conditions simultaneously under the Generalized Method of Moment (GMM)

framework. In addition, these are particularly tested to discern whether it can explain the observed high stock return volatility under recessions. This is done by using the recession-period data. Following Liu, Whited, and Zhang (2009), we find the parameters, a, b, ψ , and γ , that minimize a weighted average

of both e^{u} and $e^{\sigma^{2}}$ with one-stage GMM, in which we use the identity weighting matrix. Even though this approach can result in less efficient estimates, using identity weighting matrix equally captures the importance of asset characteristics at different time periods (Cochrane 1996). However, we also estimate the parameters by using two-stage GMM, and the estimates are similar to those from the one-stage GMM.

Variable	Definition (COMPUSTAT annual data item)
Total Debt	Short-term debt (Data 34) + long-term debt (Data 9)
Market Equity	Common shares outstanding (Data 25) \times price (Data 199)
Y (output)	Net Sales (Data 12)
i^{F} (fixed capital investment)	Capital expenditure (Data 128) – sale of PPE (Data 107)
i^{V} (variable capital investment)	(Capital invested - k^F) / N, N: time intervals per year
k^{F} (fixed asset)	Net PPE (Data 8)
k^{V} (variable asset)	(Total debt + market equity) - k^F
δ^{F} (depreciation rate of k^{F})	Average of depreciation (Data 14) to k^F over time
δ^{V} (depreciation rate of k^{V})	Value from McGrattan and Prescott (2000)
R^{E} (return on stock)	CRSP stock return data
R^{B} (return on corporate bond)	Average of Aaa and Baa bonds from Federal Reserve data
τ^{c} (corporate income tax rate)	Tax rate data from Liu, Whited, and Zhang (2009)
ω (financial leverage)	Total debt / (total debt + market equity)
heta (operating leverage)	$k^{F}_{/(}k^{F}_{+}k^{V}_{)}$

Table 1:Variable Definitions

* PPE denotes (Net) property, plant, and equipment. Monthly and quarterly COMPUSTAT flow variables are equally divided from the annual values. We also directly used quarterly COMPUSTAT data for flow variables to see seasonal effects, such as sales variations, but our results hold.

The variables in the model for the GMM estimation are from CRSP for stock returns data and COMPUSTAT for accounting data from 1974 to 2009. We exclude the financial firms (SIC codes 6000 – 6999) and regulated utilities (SIC codes 4900 – 4999) from our sample. We estimate the model based on two different time intervals: month and quarter, as well as three different time intervals overall, recession, and expansion; if the model is correct, it should hold regardless of time intervals and periods. On average, each monthly and quarterly data set incorporates more than 3,000 firms, so this data set is better than one that merely considers S&P 500 firms and is therefore sufficient to represent the overall market level. For the time alignment between COMPUSTAT variables and CRSP return and bond return data, we used 6 month lagged COMPUSTAT variables. Instead of using 6 month lagged COMPUSTAT variables, we also considered the different time alignments. For example, we

used 1 year lagged and no lagged COMPUSTAT data matching with corresponding stock and bond returns. However, the results are not sensitive. Table 1 summarizes the variable definitions.

Based on the variables above, we first estimate the associated parameters in our model. In particular, we allow time-varying adjustment costs for both fixed and variable capital investments. Fixed capital investments are less flexible than variable capital investments, and this inflexibility should be greater under recessions, since firms are much less likely to liquidate their fixed assets when the economy is bad. This causes an even higher adjustment cost. Second, we show how our model performs over two different time intervals. We further break the time period into recessions and expansions according to the U.S. business cycle from the National Bureau of Economic Research. If the results are consistently satisfactory regardless of different intervals and economic cycle periods, the model can be considered to be robust not only empirically but also theoretically. The primary measures to determine the overall model performance based on plausibly estimated parameters are: first, how well predicted returns and variations at each data point match with actual data; and second, how small the corresponding average absolute pricing errors (a.a.p.e.) are from equation (19). Third, we test which variables have significant powers to explain returns/variations and variances. Specifically, if the model's implication is correct, the financial and operating leverage should play an important role. Moreover, their roles in explaining a high variance under recessions should be especially measurable. If both leverages actually drive higher stock return volatility, a positive relation between leverages and stock returns should be observed, since investors must be compensated. With preliminary linear regression models, we test not only this association but also the amplification effect from both leverages. In addition, we conduct a comparative static analysis to measure the explanatory powers of each variable in equation (9) on errors for returns and variations. If certain variables play important roles of predicting return and variance, we should expect the model to perform worse once we remove the variation of the given variables. More importantly, we investigate how much variance changes by shutting down the variation of certain variables, which essentially measures the effect on stock return volatility. For this comparative analysis, we set the associated variable to be the average value over time. Finally, we examine the relationship between financial and operating leverages, since managers may try to stabilize the level of total leverage. This stabilization implies a trade-off between the two leverages in order to obtain the desired volatility of cash flow streams to stockholders. Instead of a single variable comparative static analysis, in this case both financial and operating leverage variables are set to be the averages over time.

Two alternative models are provided and compared in order to capture the roles of financial and operating leverages. In particular, separating fixed and variable capitals is essential in our model for the operating leverage effect. The first benchmark model is from Li and Liu (2010), in which they use tangible and intangible capital and, additionally, consider the production function of the new tangible capital investment. The characteristics of tangible and intangible capitals are in some ways similar to fixed and variable capitals. For example, plants or equipment are categorized as both tangible and fixed capitals, while R&D-related capitals are considered both intangible and variable capitals. Hence, the variables in Li and Liu (2010) may provide the proper operating leverage or at least a good proxy for it. For the second benchmark model, the difference between total invested capital and fixed assets is defined as variable assets. The ratio of the fixed assets to the total invested capital is used for the operating leverage. Except for these different sources of capital, all other aspects of the model are identical to those in the first model. In both alternative models, the gross property, plant and equipment (COMPUSTAT data item 7) for the fixed assets are used as the insensitive substitute for the net property, plant and equipment (NPPE) as in Liu, Whited, and Zhang (2009), since this group has more available data in COMPUSTAT and allows a better model estimation. Hence, this setup provides more conservative benchmark model performance. However, a better proxy for the operating leverage is the ratio of the net fixed assets to total assets as used in Mandelker and Rhee (1984) and Garcia-Feijoo and Jorgensen (2010). Our model, which is the third model, not only uses the net property, plant and equipment (COMPUSTAT data item 8) for the fixed assets even though it could diminish the model performance but also adopts the different capital accumulation process based on Cochrane (1991), in

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which he also studies the market level stock return behaviors. For both the second and third models, the variable capital investments are defined as (capital invested - k^F) divided by time intervals per year. These values match with approximately 30% of corresponding variable assets per year, which is within a reasonable range of variable capital investments to variable assets as in as in Li and Liu (2010). In fact, wide ranges of the intangible capital investments to the intangible asset s ratios are reported for the sorted portfolios in Li and Liu (2010). Because of the nature of the variable assets, which largely incorporates intangible assets, the corresponding ratios should be at least within those ranges at the market level. In unreported tests, we also consider the cases where variable capital investments per year are 40% and 50% of variable assets, and the results are not sensitive to these ranges. The operating leverage ratio is fixed to the market value of assets. The third model is the main model for this paper since its setting is more plausible and it performs better compared to the other two benchmark models.

Table 2: Means and Standard Deviations of Each Data Set

We report the summary monthly and quarterly means $(\overline{r_{t+1}^s})$ and variances $(\sigma_{r_{t+1}^s}^2)$ of stock returns based on data used for three models and two time intervals during three different periods. The GPPE and NPPE are based on the data with gross property, plant and equipment (COMPUSTAT data item 7) and with net property, plant and equipment (COMPUSTAT data item 8) for the corresponding periods, respectively. M1, M2, and M3 are the model in Li and Liu (2010), the same model with GPPE, and our model with NPPE and the capital accumulation equation in Cochrane (1991). Time periods are specified in each panel based on the U.S. business cycles from the National Bureau of Economic Research.

Panel A							
(Overall)		GPPE		NPPE			
	Μ	1	Μ	M 2		3	
	$\overline{r_{t+1}^S}$	$\sigma^2_{_{r^S_{t+1}}}$	$\overline{r_{t+1}^S}$	$\sigma^2_{r^S_{t+1}}$	$\overline{r_{t+1}^S}$	$\sigma^2_{_{r^S_{t+1}}}$	
Monthly	0.006	0.004	0.009	0.004	0.010	0.004	
Quarterly	0.011	0.015	0.033	0.017	0.041	0.015	
Panel B							
(Recessions)		GPPE			NPPE		
	Μ	[1	Μ	[2	M 3		
	$\overline{r_{t+1}^S}$	$\sigma^2_{_{r^S_{t+1}}}$	$\overline{r_{t+1}^S}$	$\sigma^2_{r^S_{t+1}}$	$\overline{r_{t+1}^S}$	$\sigma^2_{_{r^S_{t+1}}}$	
Monthly	0.004	0.009	0.009	0.008	0.004	0.008	
Quarterly	-0.039	0.028	0.072	0.036	0.005	0.031	
Panel C							
(Expansions)		GPPE			NPPE		
	Μ	[1	Μ	[2	Μ	[3	
	$\overline{r_{t+1}^S}$	$\sigma^2_{_{r^S_{t+1}}}$	$\overline{r_{t+1}^{s}}$	$\sigma^2_{\scriptscriptstyle r^S_{t+1}}$	$\overline{r_{t+1}^S}$	$\sigma^2_{\scriptscriptstyle r^S_{t+1}}$	
Monthly	0.007	0.004	0.009	0.003	0.016	0.003	
Quarterly	0.018	0.013	0.024	0.014	0.047	0.012	

Table 2 provides monthly and quarterly means and standard deviations of stock returns based on three different models and intervals during three different periods. The most noticeable feature in Table 2 is that the stock return volatilities under recessions are higher compared to other periods in all cases. For example, the monthly and quarterly stock return volatilities under the recession with the main model are more than twice as high as those under expansion, which is consistent with our model's prediction.

Table 3 reports the descriptive statistics for key variables, which will be used in our model estimation. In particular, the financial leverage ratios, W_t , are higher under recessions regardless of

models and time periods. This characteristic is consistent with the asymmetric drops in stock and bond prices during recessions. The financial leverage ratios are generally similar since we use the same definitions for all cases. However, the proxies for the operating leverage effect are measurably different among models since different definitions are used. Moreover, the first benchmark model does not seem to capture a higher operating leverage under recessions due to its less sophisticated proxy for variable assets. Under recessions, a higher adjustment cost of fixed assets implies relatively more decrease in variable assets to meet correspondingly lower market demands. This mechanism, in turn, predicts a higher operating leverage during recessions. The levels of each operating leverage ratio in the second benchmark model and the main model are significantly different since different measures of fixed assets are used. However, the ratios are comparably higher under recessions in both models. Hence, direct use of fixed assets to measure operating leverages seems more appropriate, which is as it should be. Overall, we can clearly see that both financial and operating leverages measurably increase during recessions, which provides the possible source of the higher corresponding stock return volatility as shown in Table 2.

4. Empirical Results

We first estimate the parameters for the second alternative and our models based on actual data in Table 3 by using the aforementioned GMM, and conclude that the parameters are reasonable. We did not report the results for the first alternative model here for brevity. In general, the first model does not perform well compared to the other two models. In particular, the key parameters in our model are those associated with adjustment costs. We expect a higher adjustment cost for the fixed capital, which implies a correspondingly higher parameter, as compared to the parameter for the variable capital. For example, the costs related to demolition of plants or buildings are higher than any decrease in R&D expenses, which are capitalized as variable assets and reduce the level of the variable assets. In addition, a higher parameter value associated with the fixed capital adjustment cost is expected under recession periods since liquidating fixed capital should be even more costly when the economy is bad. Based on the estimated parameters, we explicitly study the significance of the higher financial and operating leverage on a higher stock return volatility by using comparative analysis following Liu, Whited, and Zhang (2009) along with implicit tests using linear regressions.

Table 3: Descriptive Statistics for the Variables Used in Each Model

This table presents the average variables in equation (9) for overall periods given each time interval. M and Q denote the monthly and quarterly data, respectively. I and IT denote intangible and tangible, respectively. Variables with * are based on the data, in which the fixed asset is defined as net property, plant and equipment (COMPUSTAT data item 8). M1, M2, and M3 are the model in Li and Liu (2010), the same model with gross property, plant and equipment (COMPUSTAT data item 7), and our model with net property, plant and equipment (COMPUSTAT data item 7), and our model with net property, plant and equipment (COMPUSTAT data item 7). Time periods are specified based on the U.S. business cycles from the National Bureau of Economic Research.

Panel A	Overal	l (M1)		Overall (M 2)			Overall (M3)	
	М	Q		М	Q		М	Q
I_t^T / K_t^T	0.009	0.030	I_t^F / K_t^F	0.009	0.027	$I_t^{F^*} / K_t^{F^*}$	0.016	0.046
I_{t+1}^T / K_{t+1}^T	0.009	0.030	I_{t+1}^F / K_{t+1}^F	0.009	0.027	$I_{t+1}^{F^*} / K_{t+1}^{F^*}$	0.016	0.046
$(I_{t+1}^T / K_{t+1}^T) / (I_t^T / K_t^T)$	0.999	0.997	$(I_{t+1}^{F} / K_{t+1}^{F}) / (I_{t}^{F} / K_{t}^{F})$	0.999	1.001	$(I_{t+1}^{F^*} / K_{t+1}^{F^*}) / (I_t^{F^*} / K_t^{F^*})$	1.000	0.999
Y_{t+1} / K_{t+1}^T	0.133	0.361	Y_{t+1} / K_{t+1}^F	0.116	0.349	$Y_{t+1} / K_{t+1}^{F^{st}}$	0.205	0.622
$\delta^{^{T}}$	0.009	0.032	$\delta^{\scriptscriptstyle F}$	0.009	0.031	$\delta^{{\scriptscriptstyle F}^*}$	0.019	0.059
I_t^{IT} / K_t^{IT}	0.030	0.079	I_t^V / K_t^V	0.026	0.076	$I_t^{V^*}$ / $K_t^{V^*}$	0.029	0.084
I_{t+1}^{IT} / K_{t+1}^{IT}	0.030	0.079	$I_{t+1}^{V} / K_{t+1}^{V}$	0.026	0.076	$I_{t+1}^{V^*}$ / $K_{t+1}^{V^*}$	0.029	0.084
$(I_{t+1}^{IT} / K_{t+1}^{IT}) / (I_t^{IT} / K_t^{IT})$	0.998	0.999	$(I_{t+1}^V / K_{t+1}^V) / (I_t^V / K_t^V)$	0.999	0.999	$(I_{t+1}^{V^*} / K_{t+1}^{V^*}) / (I_t^{V^*} / K_t^{V^*})$	1.000	0.994
Y_{t+1} / K_{t+1}^{IT}	0.922	2.218	Y_{t+1} / K_{t+1}^{V}	0.639	1.843	$Y_{t+1} / K_{t+1}^{V^*}$	0.159	0.593
$\delta^{{}^{I\!T}}$	0.015	0.045	$\delta^{\scriptscriptstyle V}$	0.005	0.016	$\delta^{{}^{V^*}}$	0.016	0.049
r_{t+1}^B	0.006	0.018	r^B_{t+1}	0.006	0.019	r_{t+1}^B	0.007	0.028
$ au_t^c$	0.348	0.347	$ au_t^c$	0.416	0.416	$ au_t^c$	0.407	0.385
W _t	0.178	0.186	W _t	0.239	0.239	W _t	0.254	0.255
θ_{t}	0.871	0.852	θ_t	0.637	0.638	θ_{t}	0.397	0.413
I_t^T / K_t^T	0.009	0.031	I_t^F / K_t^F	0.009	0.030	$I_t^{F^*} / K_t^{F^*}$	0.017	0.051
I_{t+1}^T / K_{t+1}^T	0.009	0.030	$I_{t+1}^{F} / K_{t+1}^{F}$	0.009	0.029	$I_{t+1}^{F^{st}} / K_{t+1}^{F^{st}}$	0.017	0.051
$(I_{t+1}^T / K_{t+1}^T) / (I_t^T / K_t^T)$	0.990	0.975	$(I_{t+1}^{F} / K_{t+1}^{F}) / (I_{t}^{F} / K_{t}^{F})$	0.997	0.999	$(I_{t+1}^{F^*} / K_{t+1}^{F^*}) / (I_t^{F^*} / K_t^{F^*})$	1.002	0.997
Y_{t+1} / K_{t+1}^T	0.146	0.360	Y_{t+1} / K_{t+1}^F	0.121	0.363	$Y_{t+1} / K_{t+1}^{F^*}$	0.209	0.619
$\delta^{^{T}}$	0.010	0.032	$\delta^{\scriptscriptstyle F}$	0.010	0.031	$\delta^{{\scriptscriptstyle F}^*}$	0.019	0.059

I_t^{IT} / K_t^{IT}	0.029	0.078	I_t^V / K_t^V	0.205	0.144	$I_t^{V^*} / K_t^{V^*}$	0.034	0.111
$I_{t+1}^{IT} / K_{t+1}^{IT}$	0.029	0.078	I_{t+1}^V / K_{t+1}^V	0.212	0.145	$I_{t+1}^{V*} / K_{t+1}^{V*}$	0.035	0.116
$(I_{t+1}^{IT} / K_{t+1}^{IT}) / (I_t^{IT} / K_t^{IT})$	0.993	1.000	$(I_{t+1}^{V} / K_{t+1}^{V}) / (I_{t}^{V} / K_{t}^{V})$	1.033	1.007	$(I_{t+1}^{V^*} / K_{t+1}^{V^*}) / (I_t^{V^*} / K_t^{V^*})$	1.023	1.043
Y_{t+1} / K_{t+1}^{IT}	0.918	2.192	Y_{t+1} / K_{t+1}^{V}	5.175	3.661	$Y_{t+1} / K_{t+1}^{V^*}$	0.190	0.880
$\delta^{{\scriptscriptstyle IT}}$	0.015	0.046	$\delta^{\scriptscriptstyle V}$	0.005	0.016	$\delta^{\scriptscriptstyle V^*}$	0.016	0.050
r_{t+1}^B	0.005	0.018	r^B_{t+1}	0.006	0.021	r^B_{t+1}	0.007	0.031
$ au_t^c$	0.347	0.347	$ au_t^c$	0.411	0.413	$ au_t^c$	0.419	0.385
W _t	0.200	0.197	W _t	0.275	0.270	W _t	0.259	0.274
θ_t	0.860	0.849	θ_t	0.684	0.679	θ_t	0.419	0.446
I_t^T / K_t^T	0.009	0.030	I_t^F / K_t^F	0.008	0.026	$I_t^{F^*} / K_t^{F^*}$	0.016	0.045
I_{t+1}^T / K_{t+1}^T	0.009	0.030	$I_{t+1}^{F} / K_{t+1}^{F}$	0.008	0.026	$I_{t+1}^{F^*} / K_{t+1}^{F^*}$	0.016	0.045
$(I_{t+1}^T / K_{t+1}^T) / (I_t^T / K_t^T)$	1.001	1.000	$(I_{t+1}^{F} / K_{t+1}^{F}) / (I_{t}^{F} / K_{t}^{F})$	1.000	1.001	$(I_{t+1}^{F^*} / K_{t+1}^{F^*}) / (I_t^{F^*} / K_t^{F^*})$	0.999	1.000
Y_{t+1} / K_{t+1}^T	0.130	0.361	Y_{t+1} / K_{t+1}^F	0.115	0.348	$Y_{t+1} / K_{t+1}^{F^*}$	0.204	0.623
$\delta^{^{T}}$	0.009	0.032	$\delta^{\scriptscriptstyle F}$	0.009	0.031	$\delta^{{\scriptscriptstyle F}^*}$	0.019	0.059
I_t^{IT} / K_t^{IT}	0.030	0.079	I_t^V / K_t^V	0.026	0.060	$I_t^{V^*} / K_t^{V^*}$	0.028	0.079
$I_{t+1}^{IT} / K_{t+1}^{IT}$	0.030	0.079	$I_{t+1}^{V} / K_{t+1}^{V}$	0.025	0.060	$I_{t+1}^{V^*} / K_{t+1}^{V^*}$	0.027	0.078
$(I_{t+1}^{IT} / K_{t+1}^{IT}) / (I_t^{IT} / K_t^{IT})$	0.999	0.998	$(I_{t+1}^{V} / K_{t+1}^{V}) / (I_{t}^{V} / K_{t}^{V})$	0.954	0.996	$(I_{t+1}^{V^*} / K_{t+1}^{V^*}) / (I_t^{V^*} / K_t^{V^*})$	0.993	0.981
Y_{t+1} / K_{t+1}^{IT}	0.923	2.222	Y_{t+1} / K_{t+1}^V	0.600	1.426	$Y_{t+1} / K_{t+1}^{V^*}$	0.152	0.538
$\delta^{^{IT}}$	0.015	0.045	$\delta^{\scriptscriptstyle V}$	0.005	0.016	$\delta^{{}^{V^*}}$	0.016	0.049
r^B_{t+1}	0.006	0.018	r^B_{t+1}	0.006	0.019	r_{t+1}^B	0.006	0.028
$ au_t^c$	0.348	0.347	$ au_t^c$	0.417	0.413	$ au_t^c$	0.404	0.385
W _t	0.173	0.184	W _t	0.232	0.232	W _t	0.253	0.251
θ_t	0.873	0.852	θ_t	0.629	0.622	θ_{t}	0.392	0.406

Table 3: Descriptive Statistics for the Variables Used in Each Model - continued

4.1. Parameter Estimations and Model Performances

Table 4 presents the parameter estimates and pricing errors, which measure the models' performances. The parameter estimates from all cases show that the adjustment cost of fixed assets are greater than the adjustment costs of variable assets based on the corresponding estimated parameter values. For instance, the estimated result from our model with monthly overall data demonstrates that the parameter associated with the fixed capital adjustment cost, 7.25, is greater than the one for the variable capital adjustment cost, 4.04, as expected. Moreover, parameter values become significantly higher during recession periods as compared to expansion periods using both monthly and quarterly data. This finding supports time-varying fixed capital investment adjustment costs. When the economy is distressed, it is more difficult for firms to sell their fixed assets. Hence, this implies a higher operating leverage under recessions since firms are likelier to reduce their variable investments, which involve a relatively lower adjustment cost such that the level of variable assets decreases. This implication is confirmed in Table 3 above. Based on the second benchmark model, similar characteristics for the parameter values related to variable capital investment adjustment costs are

observed. Yet, the parameter values do not seem to vary much depending on time periods. Ψ , which captures the shape of the variable capital adjustment cost, turns out to be time-varying since the parameter values are greater or less than 1 depending on time periods. This result implies that the variable capital adjustment cost function is not necessarily convex in the investment ratio. The mean errors both for the returns and variances being zero cannot be rejected in all cases, as shown in Table 4. In fact, the average absolute pricing error is a better criterion for measuring the model performance because the mean error can average out the error to be zero by merely summing the errors. Corresponding average absolute pricing errors for returns and variations are also similar in all cases. Thus, we cannot determine a better model in between two, based on these two criteria. Specifically, using net property, plant and equipment for the net fixed asset and a different capital accumulation process for the main model, at least, do not seem to worsen the model performance.

We more precisely examine whether the model successfully predicts the returns and variations in each specific data point. Figure 1 from monthly data and Figure 2 from both monthly and quarterly data visualize each model's performance by showing how well predicted and realized returns match in general, how well they match at each data point, and how well their variations match at each data point. For the panels A in both figures, as long as each dot is close to the 45 degree line, it is implied that the model performs generally well. The main model seems to work better since the dots tend to cluster more around the 45 degree line. We also provide panels B and C showing that each predicted data set matches with each realized data set for both returns and variances at each time line under both models. Comparing between panels B demonstrates that the main model predicts each individual monthly realized return noticeably better than does the second benchmark model. Similarly, panels C show that the main model performs much better when it comes to matching with variations of each realized return data point. Since our focus is on recession periods, we reinforce the model performance by showing consistent and robust results in Figure 3, resulting from the model with monthly data under recessions. Although not presented, the main model even performs well with quarterly data during recession periods. Since the data during recession periods are not continuous, we use individual dots and stars for panels B and C at every data point instead of connecting the two. Both panels show that dots and stars are close at each point.

Table 4: Parameter Estimates and Model Performances

Parameters in panel A are estimated from one-stage GMM with an identity weighting matrix with corresponding *t*-statistics in brackets. *mean e. (1)* and *mean e. (2)* denote average errors for stock returns and variations, respectively, and associated *t*-statistics are reported in brackets. *a.a.p.e.* (1) is average absolute pricing errors for stock returns and *a.a.p.e.* (2) is average absolute pricing errors for stock returns and *a.a.p.e.* (2) is average absolute pricing errors for stock returns and *a.a.p.e.* (2) is average absolute pricing errors for stock returns and *a.a.p.e.* (2) is average absolute pricing errors for stock returns and *a.a.p.e.* (2) is average absolute pricing errors for stock return variations. *a.a.p.e.* is in percentage. M denotes the monthly and, M2, and M3 are the model in Li and Liu (2010) with gross property, plant and equipment (COMPUSTAT data item 7), and our model with net property, plant and equipment

(COMPUSTAT data item 8) and the capital accumulation equation in Cochrane (1991). Time periods are specified based on the U.S. business cycles from the National Bureau of Economic Research.

Donal A	Overall			Recessions			Expans	sions
Panel A	M 2	M 3		M 2	M 3		M 2	M 3
a	13.77	7.25	a	12.27	31.75	a	8.80	5.01
<i>(t)</i>	(0.08)	(10.54)	(<i>t</i>)	(0.03)	(0.03)	<i>(t)</i>	(1.15)	(0.02)
b	3.22	4.04	b	17.35	2.22	b	10.70	4.06
<i>(t)</i>	(0.42)	(0.28)	<i>(t)</i>	(0.61)	(87.20)	<i>(t)</i>	(0.03)	(103.44)
Ψ	1.92	1.71	Ψ	1.92	0.03	Ψ	4.19	1.68
(<i>t</i>)	(2.80)	(96.60)	(<i>t</i>)	(1.08)	(14.02)	(<i>t</i>)	(3.60)	(85.81)
γ	0.39	0.09	γ	0.58	2.00	γ	0.28	0.42
<i>(t)</i>	(3.24)	(31.97)	<i>(t)</i>	(0.39)	(3.47)	<i>(t)</i>	(3.00)	(47.72)
<i>mean e.</i> (1)	0.00	0.00	<i>mean e.</i> (1)	0.00	0.00	<i>mean e.</i> (1)	0.00	0.00
(<i>t</i>)	(0.00)	(0.00)	(<i>t</i>)	(0.00)	(0.00)	(<i>t</i>)	(0.00)	(0.00)
<i>mean e.</i> (2)	0.00	0.00	<i>mean e.</i> (2)	0.00	0.00	<i>mean e</i> . (2)	0.00	0.00
(<i>t</i>)	(0.00)	(0.00)	(<i>t</i>)	(0.00)	(0.00)	(<i>t</i>)	(0.00)	(0.00)
a.a.p.e. (1)	4.98	6.74	a.a.p.e. (1)	8.26	10.01	a.a.p.e. (1)	4.50	5.95
a.a.p.e. (2)	0.72	0.56	a.a.p.e. (2)	1.43	1.07	a.a.p.e. (2)	0.59	0.44

Parameters in panel B are estimated from one-stage GMM with an identity weighting matrix with corresponding *t*-statistics in brackets. *mean e. (1)* and *mean e. (2)* denote average errors for stock returns and variations, respectively, and associated *t*-statistics are reported in brackets. *a.a.p.e.* (1) is average absolute pricing errors for stock returns and *a.a.p.e.* (2) is average absolute pricing errors for stock returns and *a.a.p.e.* (2) is average absolute pricing errors for stock return variations. *a.a.p.e.* is in percentage. Q denotes the quarterly data, and M3 are our model with net property, plant and equipment (COMPUSTAT data item 8) and the capital accumulation equation in Cochrane (1991). Time periods are specified based on the U.S. business cycles from the National Bureau of Economic Research.

Donal D	Overall		Recessions		Expansions
r aller D	M 3 - Q		M 3 - Q		M 3 - Q
a	12.58	a	28.29	a	7.48
(<i>t</i>)	(7.35)	(<i>t</i>)	(1.01)	(<i>t</i>)	(4.81)
b	4.04	b	3.26	b	3.38
(<i>t</i>)	(179.29)	(<i>t</i>)	(6.68)	(<i>t</i>)	(25.03)
ψ	0.26	ψ	0.20	ψ	0.19
(<i>t</i>)	(21.30)	(<i>t</i>)	(5.57)	(<i>t</i>)	(9.30)
γ	2.04	γ	0.91	γ	2.30
(<i>t</i>)	(4.13)	(<i>t</i>)	(5.05)	(<i>t</i>)	(3.01)
<i>mean e.</i> (1)	0.00	<i>mean e.</i> (1)	0.00	<i>mean e.</i> (1)	0.00
(t)	(0.00)	(<i>t</i>)	(0.00)	(<i>t</i>)	(0.00)
<i>mean e</i> . (2)	0.00	<i>mean e.</i> (2)	0.00	<i>mean e.</i> (2)	0.00
(t)	(0.00)	(<i>t</i>)	(0.00)	(<i>t</i>)	(0.00)
a.a.p.e. (1)	7.11	a.a.p.e. (1)	8.86	a.a.p.e. (1)	6.93
a.a.p.e. (2)	1.65	a.a.p.e. (2)	3.02	a.a.p.e. (2)	1.28



Figure 1: The Performances of the 2nd Benchmark Model (Monthly Data)

Figure 2.1: The Performances of the Main Model (Monthly Data)





Figure 2.2: The Performances of the Main Model (Quarterly Data)

Figure 3: The Performances of the Main Model during Recessions (Monthly Data)



Unsurprisingly, predicted mean returns and variances match well with realized data in Table 1. We report the predicted monthly and quarterly means and standard deviations in Table 5. In the same table, the predicted means and standard deviations of the returns on fixed and variable assets are also provided. As we show in section 2, a higher return on the fixed assets relative to the variable assets is the necessary condition to obtain positive relations between both leverages and stock returns. These conditions are confirmed as reported in Table 5 because the means of the returns on fixed assets are generally higher. In addition, Figure 4 provides the returns on fixed and variable assets and shows the different levels of each return based on monthly and quarterly data. Thus, theoretical predictions based on our model are naturally connected to empirical findings, confirming additional credibility.

Table 5:Predicted Means and Standard Deviations of Returns
The first table (Panel A) reports the means and variances of predicted monthly and quarterly stock
returns during overall, recession, and expansion periods. The second table (Panel B) presents the
means and variances of predicted monthly and quarterly returns on fixed and variable assets for the
overall periods.

Donal A	The Main Model								
Panel A	Overall		Reces	ssions		Expansions			
	$\overline{r_{t+1}^S}$	$\sigma^2_{_{r^S_{t+1}}}$	$\overline{r_{t+1}^S}$	$\sigma^2_{\scriptscriptstyle r^S_{t+1}}$	$\overline{r_t}$	<u>5</u> +1	$\sigma^2_{r^S_{t+1}}$		
Monthly	0.010	0.004	0.004	0.008	0.0	16	0.003		
Quarterly	0.041	0.015	0.005	0.031	0.0	47	0.012		
Donal D	The Main Model								
r aller D	Overall								
	$\overline{r_{t+1}^F}$		$\overline{r_{t+1}^V}$	$\sigma^2_{r^F_{t+1}}$		$\sigma^2_{r_{t+1}^V}$			
Monthly	0.062		-0.023	0.0001			0.006		
Quarterly	0.228		0.015	-0.116			0.034		

Thus, we argue that the main model, which uses the proper proxy variable for the operating leverage ratio and the capital accumulation process as in Cochrane (1991), can be considered the appropriate Q-theory model at least for the market-level. More importantly, the strong model performances as shown above allow us to be confident about the ability of the analysis to study the roles of financial and operating leverages on stock return volatility in general, and particularly under recessions.







Figure 4.1: Monthly Returns on Fixed Capitals and Variable Capitals - continued

Figure 4.2: Quarterly Returns on Fixed Capitals and Variable Capitals



4.2. Driving Forces of Higher Stock Return Volatilities under Recessions

We hypothesize that the main driving forces of observed higher stock return volatilities during recession periods are: a higher financial leverage, a higher operating leverage, or both. If both leverages are the significant factors that derive higher stock return volatility, which is magnified during recessions, we should see strong positive relation between both leverages and long-term overall stock returns, since a higher volatility must be compensated by a higher stock return. In addition, when both leverages increase, the cash flow streams to stockholders become more volatile; this results in the aforementioned amplification effect. If this is so, we expect a positive coefficient from the interaction terms for both leverages in a multivariate linear regression model. Hence, we preliminarily regress monthly overall stock returns not only on each leverage but also on financial leverage, operating leverage, and their interaction terms simultaneously. Table 6 reports the regression results. As expected, both financial and operating leverages are positively and mostly significantly related to both

realized and predicted returns based on the single variable regressions. In particular, the relation between the financial leverage and stock returns tends to be stronger in these regressions. Hence, a higher volatility tends to be compensated by a higher return in general if higher financial and operating leverages drive higher volatility. Figures 5-1 and 5-2 visualize these relations based on monthly data. Appendix B provides the figures based on quarterly data, and the results are similar. Multivariate regression also shows similar results for individual leverage effects based on both realized and predicted return data. However, the interaction terms, which characterize the amplification effect, are only significantly positive based on monthly realized return data.

We demonstrate the positive relation between stock returns and both leverages with reduced form models. This indirectly implies a higher stock return volatility when the economy is bad. However, three following questions should be answered in order to explicitly confirm our hypothesis: Do financial and operating leverages really drive the stock return volatility? How important are these factors as compared to others in the model? If both are significantly more important than other possible factors, does the driving force for very high stock return volatility under recessions become stronger?

Similar to Liu, Whited, and Zhang (2009), we conduct a comparative static analysis to measure how different variables in the main model affect predicted stock return and variation based on overall monthly and quarterly periods. If a variable has a strong power to explain the return and variations, the model's performance should deteriorate once we set the corresponding variable to be its average. In Table 7, we report the associated model pricing errors in terms of the average absolute pricing error for both return and variation. In addition, we provide the amount that stock return volatility changes compared to the main model's results when each variable is set to its averages. In general, the model's performance deteriorates when measured by the average absolute pricing errors in both returns and variations. The most significant change in stock return volatility occurs when variable capital investments to variable assets ratios are set to their averages over time. This result is not surprising since less capital adjustment cost of variable assets allows more frequent adjustment of the variable capital investment relative to total variable assets. For instance, R&D investments are relatively flexible compared to investment in heavy equipment or plants since one cannot buy a half of equipment although it is possible to make half of R&D investments relative to the previous year. This characteristic provides flexibility in variable capital investments over time, which results in more variations compared to the investments in fixed assets. Since investments are accumulated or capitalized, the level of total variable assets themselves becomes more volatile throughout time. Furthermore, the high variation of the ratio suggests a strong correlation with the operating leverage, due to the more volatile nature of total variable assets, which determines the operating leverage of the given total assets. Even though the comparative static analysis cannot take apart the operating leverage effect from the variable capital investment to total variable capital ratio, it nonetheless implies a stronger operating leverage effect not captured by the direct operating leverage variable.

Table 6:Single and Multiple Regressions of Realized and Predicted Monthly Returns on Financial Leverage
and Operating Leverage
This table documents the positive associations between stock returns and both financial and
operating leverages. Panel A and panel B are based on monthly and quarterly stock returns,
respectively. A, B, and C in each panel denote the regressions on the financial leverage, operating
leverage, and both, with their interaction term, respectively. F.L. is financial leverage and O.L. is
operating leverage. Corresponding *t*-statistics for each coefficient are reported in brackets. F-
statistics with their p-values in brackets for the joint significance of the coefficients are also
provided.

Donal A	Realized Return				Predicted Return		
rallel A	Α	В	С		Α	В	С
Constant	-0.03	0.00	0.05	Constant	-0.06	-0.01	-0.08
<i>(t)</i>	(-2.05)	(-0.02)	(1.21)	(<i>t</i>)	(-4.15)	(-1.98)	(-1.95)

Table 6:	Single and Multiple Regressions of Realized and Predicted Monthly Returns on Financial Leverage
	and Operating Leverage - continued

F.L.	0.17		-0.10	F.L.	0.28		0.37
(<i>t</i>)	(3.05)		(-0.58)	(<i>t</i>)	(5.21)		(2.17)
<i>O.L.</i>		0.03	-0.27	<i>O.L.</i>		0.07	0.06
<i>(t)</i>		(1.75)	(-2.32)	(<i>t</i>)		(3.86)	(0.49)
F.L.*O.L.			0.90	F.L.*O.L.			-0.22
<i>(t)</i>			(2.17)	(<i>t</i>)			(-0.54)
F	9.30	3.06	4.95	F	27.19	14.89	9.13
(p-value)	(0.00)	(0.08)	(0.00)	(p-value)	(0.00)	(0.00)	(0.00)
Adj. R^2	0.02	0.01	0.02	Adj. R^2	0.05	0.03	0.05
Ν	501	501	501	N	501	501	501
Panel B	R	ealized Retur	n	_	P	1	
	Α	В	С		Α	В	С
Constant	-0.03	0.01	0.00	Constant	-0.06	0.00	-0.24
(<i>t</i>)	(-0.83)	(0.53)	(0.01)	(<i>t</i>)	(-1.58)	(-0.20)	(-2.67)
F.L.	0.29		0.76	F.L.	0.40		1.29
(<i>t</i>)	(1.94)		(1.63)	(<i>t</i>)	(2.74)		(2.80)
<i>O.L.</i>		0.07	-0.76	<i>O.L.</i>		0.11	0.37
(<i>t</i>)		(1.24)	(-1.96)	(<i>t</i>)		(2.06)	(0.96)
F.L.*O.L.			1.34	F.L.*O.L.			-1.65
(<i>t</i>)			(1.37)	(<i>t</i>)			(-1.71)
F	3.77	1.53	2.93	F	7.49	4.24	4.26
(p-value)	(0.05)	(0.22)	(0.04)	(p-value)	(0.01)	(0.04)	(0.01)
Adj. R^2	0.02	0.01	0.04	Adj. R^2	0.05	0.02	0.07
N	131	131	131	N	131	131	131

Figure 5.1: Scatter Plots with Corresponding Regression Lines (Monthly Data)

Predicted and realized monthly stock returns versus financial leverages with their corresponding regressions lines are visualized. The black line in each figure is the regression line.



Figure 5.2: Scatter Plots with Corresponding Regression Lines (Monthly Data)

Predicted and realized monthly stock returns versus operating leverages with their corresponding regressions lines are visualized. The black line in each figure is the regression line.



Table 7: Average Absolute Pricing Errors for Returns and Variations from Comparative Static Analysis and Associated Changes in Volatility M and Q denote the monthly and quarterly data, respectively. *a.a.p.e.* (1) is average absolute pricing errors for stock returns and *a.a.p.e.* (2) is average absolute pricing errors for stock return variations. Associated stock return volatility changes compared to one without variable restrictions, $\left|\Delta\sigma_{r_{el}^{2}}^{2}\right|$, are provided. Both *a.a.p.e.* and $\left|\Delta\sigma_{r_{el}^{2}}^{2}\right|$ are in percentage. For each comparative static analysis, corresponding variables are set to be their average values over time.

	Returns		Varia	ations	Change in $\sigma^2_{r_{t+1}^s}$		
	Μ	Q	Μ	Q	Μ	Q	
	a.a.p.e. (1)	a.a.p.e. (1)	a.a.p.e. (2)	a.a.p.e. (2)	$\left \Delta\sigma^2_{_{r^S_{t+1}}} ight $	$\Delta\sigma^2_{r^S_{t+1}}$	
$\overline{I_t^F/K_t^F}, \ \overline{I_{t+1}^F/K_{t+1}^F}$	6.74	7.00	0.56	1.64	0.02	-0.28	
$\overline{I_t^V / K_t^V}, \ \overline{I_{t+1}^V / K_{t+1}^V}$	4.79	9.08	0.41	1.45	97.41	93.29	
$\overline{Y_{t+1} / K_{t+1}^F}$	6.76	7.34	0.56	1.69	0.04	4.53	
$\overline{Y_{t+1} / K_{t+1}^V}$	6.76	8.49	0.57	1.80	2.65	15.88	
$\overline{W_t}$	6.82	7.44	0.59	1.83	7.90	11.93	
$\overline{ heta_t}$	6.67	11.50	0.50	2.88	14.16	75.86	
$\overline{w_t}, \ \overline{\theta_t}$	6.69	11.04	0.51	2.57	12.02	57.26	

The second and third significant changes in volatility are, surprisingly, deriving from the level of operating and financial leverages, respectively. About 8% and 14% of stock return volatility is

removed when variations of financial and operating leverages are shut down over time. This result supports our hypothesis regarding the roles of both leverages on stock return volatility. In addition, the relatively weak association between financial leverage and stock return volatility explains the weak relation between financial leverage and stock return as in Campbell, Hilscher, and Szilagyi (2008) and Garcia-Feijoo and Jorgensen (2010). However, the change in volatility when both leverages are set to their averages is less than the sum of the individual changes. This result suggests a trade-off between both leverage is set to be its average, the other leverage should be relatively high or low depending on a desired level of total leverage. As the other leverage is also set to be its average, the deviation from the desired level of total leverage becomes smaller. Thus, a lower change in stock return volatility is expected, due to the trade-off between two leverages as shown in Table 7.

First, two questions are answered. The results show that the operating leverage is the main component that explains the stock return volatility among other variables. We know how much (and how significantly) higher stock return volatility is during recessions as shown in Table 2. The third question is: why? In order to answer this last and most important question, we experiment with the same analysis using the data under recessions. The results are summarized in Table 8. Similar results are observed as in Table 7, but all the results are magnified during recessions, and the third question is also answered. The most important variable is the operating leverage and the variable capital investment to total variable assets ratio, which is essentially associated with the operating leverage for the aforementioned reason. In addition, the model deteriorates further once the variables associated with the operating leverage are set to their averages. Notably, about 33% of the stock return volatility disappears as the variations of the operating leverage variables are removed. Therefore, we argue that the primary component of higher stock return volatility under recessions is the operating leverage. The role of financial leverage is secondary.

Even though the results show that the operating leverages in conjunction with the financial leverage seem to result in stock return volatility (particularly under recessions), we must take this with a caveat. The Q-theory based stock return in the model would be simultaneously determined by all of the key variables in Table 3. Hence, setting a certain variable to be its average does not guarantee that other variables are still not affected. However, the comparative static analysis provides the mechanism for determining the driving forces on an especially higher stock return volatility under recessions, at least to some extent.

 Table 8:
 Average Absolute Pricing Errors for Returns and Variations from Comparative Static Analysis and Associated Changes in Volatility under Recessions

M and Q denote the monthly and quarterly data during recession periods, respectively. The recession periods are based on the U.S. business cycles from National Bureau of Economic Research. *a.a.p.e.* (1) is the average absolute pricing errors for stock returns and *a.a.p.e.* (2) is the average absolute pricing errors for stock return variations. Associated stock return volatility changes compared to one without variable restrictions, $\left|\Delta\sigma_{r_{\rm rel}}^2\right|$, are provided. Both *a.a.p.e.* and $\left|\Delta\sigma_{r_{\rm rel}}^2\right|$ are in percentage. For each comparative static analysis, corresponding variables are set to be their average values over time.

	Returns		Varia	ations	Change in $\sigma_{r_{t+1}^s}^2$		
	Μ	Q	Μ	Q	Μ	Q	
	a.a.p.e. (1)	a.a.p.e. (1)	a.a.p.e. (2)	a.a.p.e. (2)	$\Delta\sigma^2_{r^S_{t+1}}$	$\left \Delta\sigma^2_{r^S_{t+1}} ight $	
$\overline{I_t^F / K_t^F}, \ \overline{I_{t+1}^F / K_{t+1}^F}$	10.00	8.83	1.07	2.94	0.03	3.44	
$\overline{I_t^V / K_t^V}, \ \overline{I_{t+1}^V / K_{t+1}^V}$	7.14	13.69	0.77	2.95	94.09	90.47	
$\overline{Y_{t+1} / K_{t+1}^F}$	10.00	9.58	1.07	3.18	1.25	4.74	

 Table 8:
 Average Absolute Pricing Errors for Returns and Variations from Comparative Static Analysis and Associated Changes in Volatility under Recessions - continued

$\overline{Y_{_{t+1}} / K_{_{t+1}}^{_{V}}}$	10.02	12.40	1.07	3.64	0.69	14.54
$\overline{W_t}$	10.22	9.75	1.16	3.46	10.99	12.10
$\overline{ heta_t}$	9.17	15.35	0.84	4.23	33.27	37.07
$\overline{w_t}, \ \overline{ heta_t}$	9.20	14.61	0.85	3.91	29.90	21.58

4.3. Value Premium

Our model implies that market-level adverse economic shocks such as recessions cause both higher operating and higher financial leverage to the average firm. When such an economic downturn is expected, the average firm decreases the level of variable assets in order to deal with future decrease in demand for its output. For example, the firm can decrease its R&D expenses, which are not immediately necessary for output production. Disinvestment of variable capitals is much more flexible compared to the liquidation of fixed assets – at any time, but especially under recessions. This action by the average firm increases operating leverage, which causes equity price drops due to the additional volatility of the firm's income streams. Relatively higher equity price drop compared to bond price implies a higher financial leverage. Then we may expect abnormally high average stock return volatility.

Under recessions, this characteristic should be more pronounced in firms that normally retain high operating and financial leverages. Based on previous literature, value firms tend to maintain relatively higher financial and operating leverages than growth firms. The value firms are less likely to possess riskier assets due to the nature of fixed assets, and this implies a lower probability of incurring the costs of financial distress. Hence, the value firms have more cushion to balance their total risk, since they keep their financial leverage higher based on the trade-off theory. Precisely the opposite relation is applied to growth firms. However, the value firms' relatively high level of fixed asset causes correspondingly less flexibility. This is distinctly related to a higher adjustment cost in our model, when those firms implement downsizing in bad economic times. Hence, more of their relatively flexible variable assets are disinvested and can further increase operating leverage, which in turn results in a higher volatility of cash flow streams to stockholders. Moreover, asymmetric changes in stock and bond price during recession periods make the financial leverages of value firms increase further as well. This change causes an even higher volatility of cash flow streams or stock returns. Therefore, a higher risk premium is required for value firms since investors are much less likely to prefer holding value stocks, especially when the economy is bad. This requirement implies a reduction in stock prices and a higher stock return over time. Recently, Garcia-Feijoo and Jorgensen studied this value premium associated with, and their evidence is consistent with our model predictions. However, they use sensitivity measures for both leverages with reduced form regression models. One may use our structural form approach at firm levels to test the value premium.

5. Conclusion

In this paper, we explore the fundamental driving forces of high stock return volatility under recessions. Our parsimonious production based asset pricing model provides qualitative implications to clarify the importance of the interaction between the financial leverage effect and the operating leverage effect. This allows us to comprehend the unusually high stock return volatility we see under recessions. The increasing levels of operating leverage, due to asymmetric decreases in fixed and variable assets under recessions, cause the increasing stock return volatility and resulting immediate

stock price drops. In turn, this stock price drop relative to bond price due to anticipated uncertainty of future payoffs amplifies the stock return volatility.

Empirical results are in favor of both the financial and operating leverage as contributing factors. However, the operating leverage is among the most crucial elements that results in a higher stock return volatility during recessions. This finding explains the positive association between stock returns and both leverages over time, which many empirical studies have observed. Nonetheless, further research is necessary to confirm the causality between stock return and volatility, linked by leverages. At firm levels, this mechanism provides a source of value premium.

We offer several extensions of the study for consideration. First, one might utilize the framework of this paper to explain the value firm premium at firm levels. Value firms are more sensitive to economic shock, since they tend to have larger portion of fixed assets and debts, and this procyclical risk requires a correspondingly higher premium. Such high sensitivity to the business cycle could capture the value premium, and therefore we may expect the operating leverage to play a more significant role. Second, studying whether or not an optimal level of operating leverage exists -- particularly at firm levels -- could be intriguing. If it indeed exists, the value premium should be higher not only for the firm with a higher operating leverage but also for the firm with greater deviation from its optimal operating leverage. Finally, considering a default risk on corporate bonds will provide a theoretical robustness, since investors' expectations about firms' default probabilities are countercyclical.

Acknowledgement

We thank Gene Lai, Nathan Walcott, Sheen Liu, and seminar participants at Washington State University for their helpful comments. This work was supported by the faculty research program 2012 of Kookmin University in Korea.

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Appendix

Appendix A: Proof of Equations (7) - (9)

Following the approach in Li and Liu (2010), a firm maximizes the after-dividend firm value at each time period by optimally determining the i_{jt}^{F} , i_{jt}^{V} , k_{jt+1}^{F} , k_{jt+1}^{V} , b_{jt+1} , and l_{jt} under the constraints of equations (2) and (3) above. Specifically,

$$\begin{split} &\underbrace{\underset{i_{jt}^{F}, i_{jt}^{V}, k_{jt+1}^{F}, l_{jt} and b_{jt+1}}{Max} v_{jt} \\ s.t. \ &k_{jt+1}^{F} = (1 - \delta_{jt}^{F}) \Big[k_{jt}^{F} + i_{jt}^{F} - \Phi_{jt}^{F} \Big] \\ &k_{jt+1}^{V} = (1 - \delta_{jt}^{V}) \Big[k_{jt}^{V} + i_{jt}^{V} - \Phi_{jt}^{V} \Big] \end{split}$$

Let q_{jt}^{F} and q_{jt}^{V} are the Lagrangian multipliers of the fixed and variable capital accumulation processes, respectively. Each multiplier can represent the shadow prices for fixed and variable assets, thus the after-dividend market value of the firm's fixed assets can be defined as $q_{jt}^{F}k_{jt+1}^{F}$. Similarly, the after-dividend market value of the firm's variable asset can be defined as $q_{jt}^{V}k_{jt+1}^{V}$. This property along with equation (6) above, $v_{jt} = p_{jt} - d_{jt}^{E} + b_{jt+1}$, suggests that the total after-dividend market value of a firm at the end of period t can be expressed as

$$v_{jt} = q_{jt}^F k_{jt+1}^F + q_{jt}^V k_{jt+1}^V = p_{jt} - d_{jt}^E + b_{jt+1}$$
(A1)

The first order conditions of a firm's maximization problem with respect to k_{jt+1}^F , i_{jt}^F , l_{jt} , k_{jt+1}^V , i_{jt}^V , and b_{jt+1} provide the following equations.

$$q_{jt}^{F} = \frac{1 + (1 - \tau_{jt}^{c}) \left(\frac{\partial \Phi_{jt+1}^{F}}{\partial i_{jt+1}^{F}} \right) - \tau_{jt}^{c} \delta_{t}^{F}}{(1 - \delta_{t}^{F}) \left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial i_{jt+1}^{F}} \right)} = E_{t} \left[M_{t+1} \left(\frac{\partial v_{jt+1}}{\partial k_{jt+1}^{F}} \right) \right],$$

$$q_{jt}^{V} = \frac{1 + (1 - \tau_{jt}^{c}) \left(\frac{\partial \Phi_{jt+1}^{V}}{\partial i_{jt+1}^{V}} \right)}{(1 - \delta_{t}^{V}) \left(1 - \frac{\partial \Phi_{jt+1}^{V}}{\partial i_{jt+1}^{V}} \right)} = E_{t} \left[M_{t+1} \left(\frac{\partial v_{jt+1}}{\partial k_{jt+1}^{V}} \right) \right],$$

$$(A2)$$

$$\sigma_{jt} = \frac{\partial y_{jt}}{\partial l_{jt}}, \text{ and } E_{t} \left[M_{t+1} \left[r_{jt+1}^{b} - (r_{jt+1}^{b} - 1) \tau_{jt+1}^{c} \right] \right] = 1$$

The derivatives of the after-dividend market value function with respect to k_{jt+1}^{F} and k_{jt+1}^{V} under the restrictions of capital accumulation processes are:

$$\left\{ \frac{\partial v_{jt+1}}{\partial k_{jt+1}^{F}} = (1 - \tau_{jt+1}^{c}) \left(\frac{\partial y_{jt+1}}{\partial k_{jt+1}^{F}} - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}} \right) + \tau_{jt+1}^{c} \delta_{jt+1}^{F} + q_{jt+1}^{F} (1 - \delta_{jt+1}^{T}) \left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}} \right) \right. \\ \left. \frac{\partial v_{jt+1}}{\partial k_{jt+1}^{V}} = (1 - \tau_{jt+1}^{c}) \left(\frac{\partial y_{jt+1}}{\partial k_{jt+1}^{V}} - \frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}} \right) + q_{jt+1}^{V} (1 - \delta_{jt+1}^{T}) \left(1 - \frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}} \right) \right.$$
(A3)

Based on (A2),

$$q_{jt}^{F}k_{jt+1}^{F} + q_{jt}^{V}k_{jt+1}^{V} = E_{t} \left[M_{t+1} \left(\frac{\partial v_{jt+1}}{\partial k_{jt+1}^{F}} k_{jt+1}^{F} + \frac{\partial v_{jt+1}}{\partial k_{jt+1}^{V}} k_{jt+1}^{V} \right) \right]$$
(A4)

Let $\left(\frac{\partial v_{jt+1}}{\partial k_{jt+1}^F}k_{jt+1}^F + \frac{\partial v_{jt+1}}{\partial k_{jt+1}^V}k_{jt+1}^V\right)$ in equation (A4) define as φ_{jt+1} . Then, it can be rewritten based

on (A3) as

$$\begin{split} \varphi_{jt+1} &= \frac{\partial v_{jt+1}}{\partial k_{jt+1}^{F}} k_{jt+1}^{F} + \frac{\partial v_{jt+1}}{\partial k_{jt+1}^{V}} k_{jt+1}^{V} \\ &= \left[(1 - \tau_{jt+1}^{c}) \left(\frac{\partial y_{jt+1}}{\partial k_{jt+1}^{F}} - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}} \right) + \tau_{jt+1}^{c} \delta_{jt+1}^{F} + q_{jt+1}^{F} (1 - \delta_{jt+1}^{T}) \left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}} \right) \right] k_{jt+1}^{F} \\ &+ \left[(1 - \tau_{jt+1}^{c}) \left(\frac{\partial y_{jt+1}}{\partial k_{jt+1}^{V}} - \frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}} \right) + q_{jt+1}^{V} (1 - \delta_{jt+1}^{T}) \left(1 - \frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}} \right) \right] k_{jt+1}^{V} \end{split}$$
(A5)

By incorporating the first order conditions in (A2) and the characteristics of the constant return to scale of the production function and the investment adjustment function, φ_{jt+1} can be further simplified as

$$\varphi_{jt+1} = (1 - \tau_{jt+1}^{c})(y_{jt+1} - \overline{\omega}_{t+1}l_{jt+1} - \Phi_{jt+1}^{F} - \Phi_{jt+1}^{V}) - i_{jt+1}^{F} - i_{jt+1}^{V} + \tau_{jt+1}^{c} \delta_{jt+1}^{F}(k_{jt+1}^{F} + i_{jt+1}^{F}) + q_{jt+1}^{F}k_{jt+2}^{F} + q_{jt+1}^{V}k_{jt+2}^{V}$$
(A6)

where the term $(1 - \tau_{jt+1}^c)(y_{jt+1} - \overline{\omega}_{t+1}l_{jt+1} - \Phi_{jt+1}^F - \Phi_{jt+1}^V) - i_{jt+1}^F - i_{jt+1}^V + \tau_{jt+1}^c \delta_{jt+1}^F (k_{jt+1}^F + i_{jt+1}^F)$ in (A6) implies

the free cash flow of the firm at time t+1, which is d_{jt+1}^E . From equation (A1), $q_{jt+1}^F k_{jt+2}^F + q_{jt+1}^V k_{jt+2}^V$ in equation (A6) is v_{jt+1} such that $\varphi_{jt+1} = d_{jt+1}^E + v_{jt+1}$. Hence, by iteration

$$q_{jt}^{F}k_{jt+1}^{F} + q_{jt}^{V}k_{jt+1}^{V} = E_{t}\left[M_{t+1}\left[d_{jt+1}^{E} + v_{jt+1}\right]\right] = E_{t}\left[\sum_{s=1}^{\infty} M_{t+s}d_{jt+s}^{E}\right]$$
(A7)
$$E_{t}\left[M_{t+1}\left[r_{jt+1}^{b} - (r_{jt+1}^{b} - 1)\tau_{jt+1}^{c}\right]\right] = 1$$
in constant

in

From

since $d_{jt+1}^{E} + p_{jt+1} + \lfloor r_{jt+1}^{b} - r_{jt+1}^{b} (1 - \tau_{jt+1}^{c}) \rfloor b_{jt+1} = d_{jt+1}^{E} + p_{jt+1} + b_{jt+2}$. Therefore, the relations in equation (A1) are confirmed.

From (A9), $q_{jt}^{F}k_{jt+1}^{F} + q_{jt}^{V}k_{jt+1}^{V} = E_{t}\left[M_{t+1}\left[d_{jt+1}^{E} + v_{jt+1}\right]\right] = E_{t}\left[M_{t+1}\left[d_{jt+1}^{E} + q_{jt+1}^{F}k_{jt+2}^{F} + q_{jt+1}^{V}k_{jt+2}^{V}\right]\right]$ which implies

$$1 = E_{t} \left[M_{t+1} \left[\frac{d_{jt+1}^{E} + q_{jt+1}^{F} k_{jt+2}^{F} + q_{jt+1}^{V} k_{jt+2}^{V}}{q_{jt}^{F} k_{jt+1}^{F} + q_{jt}^{V} k_{jt+1}^{V}} \right] \right]$$

$$\frac{d_{jt+1}^{E} + q_{jt+1}^{F} k_{jt+2}^{F} + q_{jt+1}^{V} k_{jt+2}^{V}}{E^{LE} + \frac{V}{2} V}$$
(A11)

 $q_{it}^{F}k_{it+1}^{F} + q_{jt}^{V}k_{jt+1}^{V}$ in equation (A11) is, by definition, the firm's total return on assets, r_{jt+1}^{A} Then $E_{t}\left[M_{t+1}r_{jt+1}^{A}\right] = 1$. In addition,

$$\frac{d_{jt+1}^{E} + q_{jt+1}^{F}k_{jt+2}^{F} + q_{jt+1}^{V}k_{jt+2}^{V}}{q_{jt}^{F}k_{jt+1}^{F} + q_{jt}^{V}k_{jt+1}^{V}} = \frac{d_{jt+1}^{E} + \left[r_{jt+1}^{b} - \tau_{jt+1}^{c}(r_{jt+1}^{b} - 1)\right]b_{jt+1} - b_{jt+2} + v_{jt+1}}{v_{jt}}$$

$$= \frac{d_{jt+1}^{E} + r_{jt+1}^{b}b_{jt+1} - b_{jt+2} + p_{jt+1} - d_{jt+1}^{E} + b_{jt+2}}{v_{jt}}$$

$$= \frac{p_{jt+1} + r_{jt+1}^{b}b_{jt+1}}{v_{jt}} = (1 - w_{t})r_{jt+1}^{E} + w_{t}r_{jt+1}^{b}$$
(A12)

From the first order condition in (A2) in terms of the human capital l_{jt} , $\boldsymbol{\sigma}_{jt} = (1-\alpha) \left[\left(k_{jt}^{F} \right)^{\gamma} \left(k_{jt}^{V} \right)^{1-\gamma} \right]^{\alpha} \left[A_{jt} l_{jt} \right]^{-\alpha}$. Then the following equation is obtained, $y_{jt} - \boldsymbol{\sigma}_{jt} l_{jt} = y_{jt} - \left[(1-\alpha) \left[\left(k_{jt}^{F} \right)^{\gamma} \left(k_{jt}^{V} \right)^{1-\gamma} \right]^{\alpha} \left[A_{jt} l_{jt} \right]^{-\alpha} \right] l_{jt} = \alpha y_{jt}$ (A13) $d^{E} + \alpha^{F} k^{F} + \alpha^{V} k^{V}$

In addition, by substituting equation (2), (3), (A2), and (A13) into $\frac{d_{jt+1}^{E} + q_{jt+1}^{F} k_{jt+2}^{F} + q_{jt+1}^{V} k_{jt+2}^{V}}{q_{jt}^{F} k_{jt+1}^{F} + q_{jt}^{V} k_{jt+1}^{V}}$

and dividing both the numerator and denominator by k_{jt+1}^{F} ,

$$r_{jt+1}^{A} = \frac{\left(1 - \tau_{jt+1}^{c}\right) \left(\frac{\gamma y_{jt+1}}{k_{jt+1}^{F}}\right) - (1 - \tau_{jt+1}^{c}) \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}} + \tau_{jt+1}^{c} \delta_{jt+1}^{F} + q_{jt+1}^{F} (1 - \delta_{jt+1}^{F}) \left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}}\right)}{\left(1 - \frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}}\right) - (1 - \tau_{jt+1}^{c}) \frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}}\right] \left(\frac{k_{jt+1}^{V}}{k_{jt+1}^{F}}\right)}{\left(1 - \tau_{jt}^{c}\right) \left(\frac{\partial \Phi_{jt+1}^{F}}{\partial i_{jt+1}^{F}}\right) - \tau_{jt}^{c} \delta_{t}^{F}}{\left(1 - \delta_{t}^{V}\right) \left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial i_{jt+1}^{F}}\right)}\right] + \left(\frac{1 + (1 - \tau_{jt}^{c}) \left(\frac{\partial \Phi_{jt+1}^{V}}{\partial i_{jt+1}^{V}}\right)}{(1 - \delta_{t}^{V}) \left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial i_{jt+1}^{F}}\right)}\right) \left(\frac{k_{jt+1}^{V}}{k_{jt+1}^{F}}\right) - \frac{k_{jt+1}^{V}}{k_{jt+1}^{F}}\right)$$

Finally, let θ_{jt} denote the ratio of fixed asset value to total asset value such that $\begin{cases}
\theta_{jt}^{F} = \frac{q_{jt}^{F} k_{jt+1}^{F}}{v_{jt}} \\
1 - \theta_{jt}^{F} = \frac{q_{jt}^{V} k_{jt+1}^{V}}{v_{t}}
\end{cases}$

Based on equation (A1) and (A2), $r_{jt+1}^{A} = \theta_{jt}r_{jt+1}^{F} + (1-\theta_{jt})r_{jt+1}^{V}$ is satisfied if and only if

$$r_{jt+1}^{F} = \frac{(1 - \tau_{jt+1}^{c}) \left(\alpha \frac{\gamma y_{jt+1}}{k_{jt+1}^{F}} - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}} \right) + \tau_{jt+1}^{c} \delta_{jt+1}^{F} + q_{jt+1}^{F} (1 - \delta_{jt+1}^{F}) \left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial k_{jt+1}^{F}} \right)}{\left[\frac{1 + (1 - \tau_{jt}^{c}) \left(\frac{\partial \Phi_{jt+1}^{F}}{\partial i_{jt+1}^{F}} \right) - \tau_{jt}^{c} \delta_{t}^{F}}{(1 - \delta_{t}^{F}) \left(1 - \frac{\partial \Phi_{jt+1}^{F}}{\partial i_{jt+1}^{F}} \right)} \right]}$$

and

$$r_{jt+1}^{V} = \frac{(1 - \tau_{jt+1}^{c}) \left[(1 - \alpha) \frac{\gamma y_{jt+1}}{k_{jt+1}^{V}} - \left(\frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}} \right) \right] + q_{jt+1}^{V} (1 - \delta_{jt+1}^{V}) \left(1 - \frac{\partial \Phi_{jt+1}^{V}}{\partial k_{jt+1}^{V}} \right)}{\left[\frac{1 + (1 - \tau_{jt}^{c}) \left(\frac{\partial \Phi_{jt+1}^{V}}{\partial l_{jt+1}^{V}} \right)}{(1 - \delta_{t}^{V}) \left(1 - \frac{\partial \Phi_{jt+1}^{V}}{\partial l_{jt+1}^{V}} \right)} \right]}$$

, where the derivatives of investment adjustment costs with respect to the i_{jt}^F , k_{jt}^F , i_{jt}^V , and k_{jt}^V can be expressed as

$$\begin{cases} \frac{\partial \Phi_{jt}^{F}}{\partial i_{jt}^{F}} = \frac{a(\rho+1)}{2} \left(\frac{i_{jt}^{F}}{k_{jt}^{F}}\right)^{\rho} \\ \frac{\partial \Phi_{jt}^{F}}{\partial k_{jt}^{F}} = \frac{-a\rho}{2} \left(\frac{i_{jt}^{F}}{k_{jt}^{F}}\right)^{\rho+1} \\ \frac{\partial \Phi_{jt}^{V}}{\partial i_{jt}^{V}} = \frac{b(\psi+1)}{2} \left(\frac{i_{jt}^{V}}{k_{jt}^{V}}\right)^{\psi} \\ \frac{\partial \Phi_{jt}^{V}}{\partial k_{jt}^{V}} = \frac{-b\psi}{2} \left(\frac{i_{jt}^{V}}{k_{jt}^{V}}\right)^{\psi+1} \end{cases}$$

See Li and Liu (2010) for more details.

Appendix B.1: Scatter Plots with Corresponding Regression Lines (Quarterly Data)

Predicted and realized quarterly stock returns versus financial leverages with their corresponding regressions lines are visualized. The black line in each figure is the regression line.



Appendix B.2: Scatter Plots with Corresponding Regression Lines (Quarterly Data)

Predicted and realized quarterly stock returns versus operating leverages with their corresponding regressions lines are visualized. The black line in each figure is the regression line.

