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Input-Output Analysis with Linear Programming: The Case of Turkey

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Abstract

Economic planning is a vital tool for countries to achieve socio-economic goals and objectives, which can be expressed quantitatively. Input-output analysis and linear programming techniques are convenient and efficient methods for economic planning, that is why they are being used in a widespread manner. Input-output analysis is based on examining input-output table which reflects the all goods, services and the sectoral relationships in an economy particularly. Input-output model is a consistency model and it does not deal with the matter of optimal resource allocation. For that reason, there is in need of an optimization technique. In this study, Linear Programming techniques are used, because some applicable economic assumptions give way to express all the sectoral relationships linearly. In this context, it is useful to implement both Input-output analysis and Linear Programming techniques to obtain the most effective and consistent results. The purpose of this study is to determine the optimal distribution of economic resources on sectoral basis to maximize the national income. In this sense, the economic structure of Turkey is investigated and the sectoral production levels are specified with Linear Programming techniques by using the latest input-output table, which is organized by Turkish Statistical Institute for the year 2002.

Keywords: Input-Output Analysis, Linear Programming
JEL Classification Codes: C44; C61; C67; D57; R15

1. Introduction

Countries must use their economic scarce resources in planned and scheduled for maintaining their presence and development in the competition environment. Therefore, economic planning has been an essential element for economies, which was generated in 1920s and has been used widely after the Second World War. Economic planning methods show variety according to countries' economic structures. In general, the economic plans are being prepared with two approaches; single-stage planning and multi-stage planning. Single-stage planning covers all the economic relationships and requires the solution of very complex mathematical models. This approach is usually used by developed countries because of need having too much data and qualified experts for the election of variables in the model. Most of the developing countries prefer multi-stage planning, which is relatively easier than single-stage planning. In Turkey, the multi-stage planning approach has been applied since 1960s. Multi-stage planning consists of three stages; macro planning, sectoral planning and project planning. At the macro planning stage, the decisions and the objectives are specified regarding macroeconomic indicators. Sectoral planning stage serves as a bridge between macro planning stage and project planning stage, which involves the assessment of the investment projects in line with the objectives that were determined at the macro planning stage. At the sectoral planning stage, the economy is examined on a sector basis, and it is aimed to determine the levels of sectoral production to meet the intermediate demand and the final demand for their output. The major models that serve this purpose are the input-output models and the linear programming models, which are applied in many countries' economic planning process.

The input-output models developed by Wassily Leontief in the late 1930s. They are consistency models that are used to determine the sectoral product levels to ensure the supply-demand balance in an economy. However, these models can give no answer to target the most efficient use of scarce resources in the economy. Linear Programming technique can overcome this deficiency. The Linear Programming models, which are namely activity planning in literature, are being constituted from Input-Output table and used to determine the optimal levels of sectoral output to ensure achievement of the intended goals. In this sense, the first application of linear programming techniques in the field of economics was to the area of input-output analysis (Gass, 1969). The theory of input-output can also be regarded as a peculiarly simple form of linear programming: in the simplest Leontief system, in which no substitutions of inputs are technologically feasible, the optimizing solution is the one and only efficient solution possible; but in more general models, in which substitution is possible, the system can be made determinate only by solving an appropriately formulated linear programming problem (or by requiring the solution to satisfy some restrictive outside conditions) (Dorfman, Samuelson, & Solow, 1987).

The characteristic goal of economic planning is maximizing the national income under current conditions. In this study, the optimal sectoral output levels that are maximizing the national income are investigated. In the application, the latest Turkey Aggregated Input-Output table is used, which is organized by Turkish Statistical Institute for 2002 (Türkiye İstatistik Kurumu, 2008).

2. Input-Output Model

The focus on the economy as a whole gives input-output analysis a macroeconomic flavor, but its foundation and techniques are more microeconomic, including a rigorous grounding in production and consumption (Raa, 2005). Input-Output models analysis sectors and inter-sectoral relationships in an economy, which are based on Walras' abstract general equilibrium equations. In its most basic form, an input-output model consists of a system of linear equations, each one of which describes the distribution of a sector's product throughout the economy. The fundamental information used in input-output analysis concerns the flows of products from each sector, considered as a producer, to each of the sectors, itself and others, considered as consumers. This basic information from which an input-output model is developed is contained in an input-output table (Miller & Blair, 2009). Input-Output table is an essential tool that includes all production and consumption units and the flow of goods and services among these units during the preparation of the planning. From this table, it is possible to generate two basic equilibriums:

2.1. General Equilibrium Equations

X_i: The total output of ith sector

X_{ij}: The output of ith sector, used by jth sector
Y_i: The total final domestic demand of ith sector
M_i: The imports of ith sector
E_i: The exports of ith sector
L_j: The total amount of labor, used by jth sector
K_j: The total amount of capital, used by jth sector
C_i: The amount of ith output used for the consumption
I_i: The amount of ith output used for the investment
G_i: The amount of ith output used for the government expenditure

2.1.1. Row Approach (Quantity System)

The total supply *(output+imports)* of any sector i is equal to its total demand *(final demand+exports)*. This equality is called the supply-demand equilibrium, which is as shown below:

$$X_i + M_i = \sum_{j=1}^{n} X_{ij} + Y_i + E_i \qquad \text{for } i = 1,2,\ ...,\ n$$

where

$$Y_i = C_i + G_i + I_i \quad \text{for } i = 1, 2,\dots,n.$$

2.1.2. Column Approach (Equilibrium Price System)

The product value of any j sector equals to the total payment for the intermediate and final inputs. In consequence of, the equilibrium price equation can be defined as,

$$X_j = \sum_{i=1}^{n} X_{ij} + L_j + K_j \qquad \text{for } j = 1,2,\ ...,\ n.$$

Input-Output models cover the all sectors of an economy and that is why they gain a macro-economic feature. Macro-economic analysis requires the measurement of the economic activity level as a whole and national income is one of the basic concepts used for this measurement. National income can be calculated with the “expenditures” and the “factor incomes” methods by using Input-Output tables.

On the expenditures method, national income is the sum of consumption, investment and government expenditures for final goods and services.

According to factor incomes method, the total incomes of production factors give the national income. In other words, the sum of each sectors’ payments to the basic inputs *(the sum of net value added)* equals to national income.

The calculation of national income is based on the assumption of the economic equilibrium in the long term. In accordance with this assumption, the monetary value of sectors’ outputs is same as their production costs. Therefore, the national incomes, calculated by both methods are equivalent. For all sectors, this equality as follows:

$$\sum_{i=1}^{n} Y_i + \sum_{i=1}^{n} E_i - \sum_{i=1}^{n} M_i = \sum_{j=1}^{n} L_j + \sum_{j=1}^{n} K_j$$

2.2. Input Coefficients Matrix and Leontief Inverse Matrix

To explain the Leontief’s solution method, assume that the economy is a closed economy and consists of n sectors. In this case, the supply-demand equilibrium can be represented by,

$$X_j = \sum_{i=1}^{n} X_{ij} + Y_i \qquad i = 1, 2,\ \ ...,\ n.$$

If this equation is written in the open form for all sectors, a linear equation system is obtained as the following:

$$X_1 = x_{11} + x_{12} + \ldots x_{1n} + Y_1$$
$$X_2 = x_{21} + x_{22} + \ldots x_{2n} + Y_2$$
$$\vdots$$
$$X_n = x_{n1} + x_{n2} + \ldots + x_{nn} + Y_n$$

where the final demands (Y) are foreknown. So the equations can be written such that,

$$\begin{aligned} X_1 - x_{11} - x_{12} - \ldots - x_{1n} &= Y_1 \\ X_2 - x_{21} - x_{22} - \ldots - x_{2n} &= Y_2 \\ &\vdots \\ X_n - x_{n1} - x_{n2} - \ldots - x_{nn} &= Y_n \end{aligned} \tag{1}$$

This linear equation system involves $n+n^2$ unknowns and n equations. Therefore, there is not only one solution of this equation system. To obtain a single solution, an assumption can be used. This assumption asserts, "the demand of any ith output by any jth sector, is a linear function of jth sector's production level" (Can, 2006). This function is called input function and can be represented by,

$$a_{ij} = \frac{x_{ij}}{X_j}$$

for i=1,2,…,n and j=1,2,…,n..

a_{ij} is named input coefficient or technology coefficient by the reason of it reflects the technologic structure of an economy within a fixed time.

The input coefficient indicates the required minimum ith output to produce one unit jth output with the current production technique.

With rearranging the equation system (1) by using the input coefficients, the equation system is transformed into matrix form,

$$(I–A)X=Y$$

The matrix $(I – A)$ is called the Leontief matrix. By using this formula the sectoral production levels can be obtained with the following equation,

$$X=(I–A)^{-1}Y$$

This solution is the general solution of static Input-Output system and the $(I-A)^{-1}$ is called Leontief inverse matrix or the total requirements matrix.

The assumption about the input coefficient is also valid for the basic inputs as well as the intermediate inputs. The amounts of the whole labor and capital, for use by any j sector's total production, presented with L_j and K_j respectively. In that case, the required basic inputs for the production of a unit output can be obtained by using the ratio of total basic inputs to total production;

$$l_j = \frac{L_j}{X_j} \quad \text{and} \quad k_j = \frac{K_j}{X_j}$$

for j=1,2,…,n l_j specifies the minimum required labor and K_j indicates the minimum required capital to produce a unit jth output. In general, these coefficients are called factor intensity coefficients.

In an open economy, imports creates an additional supply source, however adding imports directly to the Input-Output model, causes some disadvantages. Some imported products are rival or substitute commodities to the domestic products in both intermediate and final usage. On the other hand, some other imported products are absolute complementary goods for the domestic production. In Input-Output models, the methods followed to deal with the imports, differs depending on their rival or complementary features. The commonly used method was propounded by H.B. Chenery and P.G. Clark (Chenery & Clark, 1959). This method is arose from the acceptance of the imported goods are rival commodities to domestic products. In addition to this, the underlying property of the method is; the import is a linear and constant function of domestic production for all sectors (Aydoğuş, 1999). Therefore, the sectoral import functions can be defined as the following,

$M_i=m_i x_i$ for i=1,2, ..., n

where m_i implies the import coefficient. This coefficient signifies the imports per unit of the domestic production.

3. Linear Programming Model

In terms of economics, the most important objectives of economic planning are; the optimal distribution of resources, full usage of labor and production capacity, maximizing the standard of life and maximizing the economic growth. National income is the general criterion, which involves these objectives. In planned development, it is aimed to attain the maximum national income at the end of the plan period. In this context, the model, which will be established, intends to determine the sectoral production levels that maximize the national income *(net value added)*. In the maximization process of national income, there are many inevitable restrictions in the economy. The first of these restrictions is relevant to supply-demand equilibrium. This equilibrium explains that, the total demand of any sector cannot exceed the sum of its total output and imports. Another restriction is related to basic inputs. The demand of labor cannot exceed the amount of available labor and similarly, the production levels of sectors cannot exceed the available production capacity.

Within the scope of these explanations, the linear programming model (Bozdağ & Altan, 1995) which maximizes the national income is,

$$[\mathrm{Max}]Z = \sum_{j=1}^{n} c_j x_j$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j + Y_i + E_i \le (1 + m_i) x_i$$

$$\sum_{j=1}^{n} l_j x_j \le L$$

$$\sum_{j=1}^{n} k_j x_j \le K$$

$$x_j \ge 0$$

where,

X_j: The sectoral production level of jth sector

c_j : The net value added which is derived from one unit production of jth sector (value added coefficient),

a_{ij}: The amount of ith output which is necessary to produce one unit of commodity j,

Y_i: The final domestic consumption of sector i,

E_i: The exports of ith sector,

m_i: The import coefficient of sector i,

l_j: The amount of labor, which is necessary to produce one unit of commodity j,

L: The amount of available labor in the economy,

k_j: The amount of capital, which is necessary to produce one unit of commodity j,

K: The amount of available capital in the economy.

In application, the latest aggregated Input-Output table is used, which was organized by Turkey Statistical Institute for 2002. In this table the economy is reduced to six main sectors;

1. Agriculture, hunting, forestry and fishing
2. Industry, including energy
3. Construction
4. Wholesale and retail trade, hotels and restaurants; transport and communications
5. Financial, real estate, renting and business activities
6. Other service activities

Therefore, the decision variables of the linear programming model are;

x_i: The sectoral output level of agriculture, hunting, forestry and fishing,
x_2: The sectoral output level of industry, including energy,
x_3: The sectoral output level of construction,
x_4: The sectoral output level of wholesale and retail trade, hotels and restaurants; transport and communications,
x_5: The sectoral output level of financial, real estate, renting and business activities,
x_6: The sectoral output level of other service activities
where each decision variable corresponds to a sector respectively.

The linear programming model is obtained from the aggregated Input-Output table setting as,

Objective Function

$$Z_{\max} = 0.6278\,x_1 + 0.2814\,x_2 + 0.4319\,x_3 + 0.5461\,x_4 + 0.6980\,x_5 + 0.6114\,x_6$$

Subject to:

$$C_1 : 0.9068\ x_1 - 0.3806\ x_2 - 0.0002\ x_3 - 0.0422\ x_4 - 0.0069\ x_5 - 0.0063\ x_6 \geq 23129060$$
$$C_2 : -0.0227\ x_1 + 0.8309\ x_2 - 0.0502\ x_3 - 0.0863\ x_4 - 0.0370\ x_5 - 0.0323\ x_6 \geq 128131300$$
$$C_3 : -0.0042\ x_1 - 0.0043\ x_2 + 0.9833\ x_3 - 0.0113\ x_4 - 0.0253\ x_5 - 0.0190\ x_6 \geq 1313863$$
$$C_4 : -0.0160\ x_1 - 0.1781\ x_2 - 0.01912\ x_3 + 0.8277\ x_4 - 0.0257\ x_5 - 0.0279\ x_6 \geq 85793790$$
$$C_5 : -0.0149\ x_1 - 0.1030\ x_2 - 0.0148\ x_3 - 0.1768\ x_4 + 0.9278\ x_5 - 0.0673\ x_6 \geq 44909330$$
$$C_6 : -0.0016\ x_1 - 0.0051\ x_2 - 0.0003\ x_3 - 0.0139\ x_4 - 0.0226\ x_5 + 0.9788\ x_6 \geq 54144780$$
$$C_7 : 0.0982\ x_1 + 0.1061\ x_2 + 0.1330\ x_3 + 0.1164\ x_4 + 0.0858\ x_5 + 0.5101\ x_6 \leq 92431100$$
$$C_8 : 0.0431\ x_1 + 0.0386\ x_2 + 0.0190\ x_3 + 0.0535\ x_4 + 0.0249\ x_5 + 0.0264\ x_6 \leq 25227610$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

3.1. The Optimal Solution of the Linear Programming Model

The problem is solved by WinQSB program and Table 1 shows the optimal solution of the problem. The optimal solution is achieved with ten iterations and the objective function's maximum value is determined as 340.991.400 billion TL.

Table 1: The Optimal Solution (billion TL)

Decision Variable	Solution Value	Unit Cost or Profit (c_j)	Total Contribution	Reduced Cost	Basis Status
x_1	110.589.200	0.6278	69.423.720	0	Basic
x_2	182.185.700	0.2814	51.263.610	0	Basic
x_3	9.066.759	0.4319	3.916.046	0	Basic
x_4	153.680.600	0.5461	83.923.360	0	Basic
x_5	135.666.400	0.6980	94.699.110	0	Basic
x_6	61.773.400	0.6114	37.765.510	0	Basic
Objective Function	340.991.400				

For every constraint, how much resource is used from the available resource capacity represented on Table 2. As shown under the title of "Slack or Surplus", the entire amount of available resources are used over C_1, C_2, C_3, C_4, C_6 and C_7 constraints. Nevertheless, there are idle resource capacities 29.079.450 and 12.850 billion TL on C_5 and C_8 constraints respectively.

Table 2: Constraints (billion TL)

Constraint	Left Hand Side	Right Hand Side	Slack or Surplus	Shadow Price
C_1	23.129.060	23.129.060	0	-0.0738
C_2	128.131.300	128.131.300	0	-0.6735
C_3	1.313.863	1.313.863	0	-0.5216
C_4	87.593.790	87.593.790	0	-0.4246

Table 2: Constraints (billion TL) - continued

C_5	73.988.780	44.909.330	29.079.450	0
C_6	54.144.780	54.144.780	0	-2.9549
C_7	92.431.100	92.431.100	0	6.7819
C_8	25.214.760	25.227.610	12.850	0

Shadow price of a constraint i is defined to be the rate of the change in the objective function as a result of a change in the value of the right hand side of constraint i (Dantzig & Thapa, 1997). The "Shadow Price" column (Table 2) represents each constraint's effect in the objective function in the case of one unit increasement of the right hand side of each constraint. C_7 has a positive effect and C_1, C_2, C_3, C_4, C_6 have negative effects. The changes, related to C_5 and C_8 do not create any impact.

Table 3: Sensitivity Analysis of the Objective Function's Coefficients (billion TL)

Decision Variable	Unit Cost or Profit (c_j)	Allowable Min. c_j	Allowable Max. c_j
x_1	0.6278	-23.8903	0.6957
x_2	0.2814	-3.3916	0.8692
x_3	0.4319	-2.8379	0.9627
x_4	0.5461	-3.9369	0.9088
x_5	0.6980	0.6262	∞
x_6	0.6114	-2.6961	3.8559

Sensitivity analysis (also called post optimality analysis) deals with the problem of obtaining an optimum feasible solution of the modified problem starting with the optimum feasible solution of the old problem (Murty, 1976). The allowable intervals of the objective function's coefficients and right hand sides, which do not affect the optimal solution, are given on Table 3 and Table 4.

Table 4: Sensitivity Analysis of the Right Hand Sides (billion TL)

Constraint	Right Hand Side (RHS)	Allowable Min. RHS	Allowable Max. RHS
C_1	23.129.060	-78.745.050	23.820.770
C_2	128.131.300	-30.872.660	128.646.900
C_3	1.313.863	516.253,6	25.210.790
C_4	87.593.790	-43.706.070	88.063.130
C_5	44.909.330	-∞	73.988.790
C_6	54.144.780	54.033.140	60.868.720
C_7	92.431.100	88.747.020	92.477.750
C_8	25.227.610	25.214.760	∞

Table 5 displays the comparison between the consistency and the activity planning. Results show that, increasing the sectoral product levels of agriculture, financial and other service activities and decreasing the sectoral product levels of industry, construction and wholesale and retail trade proportionately, it is possible to increase the national income at least 10%.

Table 5: The Comparison of the values 2002 aggregated Input-Output Table and the values obtained by Linear Programming analysis (billion TL)

Sectors	Consistency Planning	Activity Planning
Agriculture, hunting, forestry and fishing	51946913	110589216
Industry, including energy	250457257	182185707
Construction	32439902	9066758
Wholesale and retail trade, hotels and restaurants; transport and communications	169827364	153680593
Financial, real estate, renting and business activities	83221597	135666386

Table 5: The Comparison of the values 2002 aggregated Input-Output Table and the values obtained by Linear Programming analysis (billion TL) - continued

Other service activities	57892587	61773401
Gross Value Added	303320435	340991349

4. Conclusion

When the developed countries of European Union members is examined, it can be seen that agriculture, financial and service activities has become more important sectors and their shares are higher than the other sectors in national production.

In today's Turkey, there are policies and discourses to increase financial and service activities but agriculture is not taken into account sufficiently.

On the other hand, there is a high-unutilized capacity in financial. Similarly, a low rate of unutilized capacity occurs in capital. Service revenues and capital accumulation in rents will be expected and required feature.

In this current economic structure, one unit increment in final demands and exports (one billion TL) of agriculture, industry, construction, retail trade and other services sectors will lead to declines. The most effective sector is other service activities in these reductions.

Changing in capital and financial demands does not cause any difference in the national income. One unit augmentation in labor, enables nearly seven times more contribution in national income according to the current situation and this is the most important indicator that, Turkey's industry and manufacturing is still have labor-intensive structure.

Input-Output analysis is a consistency model and from this consistency table , how to plan the optimal distribution of economic resources in order to maximize the national income with the linear programming technique, in other words the activity planning is analyzed.

In this study, the linear programming method is used for the activity analysis. Of course, except for linear programming, different techniques such as nonlinear programming or dynamic programming methods can be used for Input-Output analysis.

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