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Modeling and Forecasting Time Varying Stock Return Volatility in the Egyptian Stock Market

Moustafa Ahmed Abd El Aal
Researcher of Finance, Alexandria University, Egypt
E-mail: mostafamba@yahoo.com

Abstract

This study investigates the performance of five models for forecasting the Egyptian stock market return volatility. We used the period from 1 January, 1998 until 31 December, 2009 as an in-sample period. We used also the next 30 days after the in-sample period to be our out-of-sample period. The competing models are: EWMA, ARCH, GARCH, GJR, and EGARCH. We examined also the ARCH effect to test the validity of using GARCH family to predict the volatility of market indices. The empirical results show that EGARCH is the best model between the examined models according to the usual evaluating statistical metrics (RMSN, MAE, and MAPE). When we used Diebold and Mariano (DM) test to examine the significance of the difference between errors of volatility forecasting models, we found no significance difference between the errors of competing models. The results also reject the null hypothesis of homoscedastic normal process for both EGX30 and CIBC100 indices.

Keywords: Volatility, ARCH effect, GARCH family, and Egyptian market.
JEL Classification Code: G17

1. Introduction

The Egyptian securities market consider one of the most important emerging markets in the Middle East because of its listed securities number, number of investors, trading volume, …. Etc. The Egyptian market hasn't any significant activity until the 1990s. The incentives for foreign investments and removing restrictions on it made the Egyptian market started growing rapidly. The IPO attracted big number of investors to the market. The world economic crisis in 2008 has a huge affection on the Egyptian market, it made EGX30 index fall from 12000 points to 3380 points

The Capital Market Authority (CMA) has introduced several procedures to improve the efficiency of the market and minimizing the risk of it such as: introducing the intra-day (T+0) system instead of T+2/3, introducing the short selling and margin trading, and introducing of Exchange Traded Funds (ETFs). On the other hand the Egyptian market is waiting for the introduction of financial Derivatives.

Thus, the study of risk management is an important issue. Study of volatility is important in risk management. There are several models for forecasting the volatility of stock market return. This study evaluates these models using three market indices; EGX30, CIBC100, HFI.

2. The Objective of the Study

This study evaluates the performance of five forecasting volatility models; these models are: EWMA, ARCH, GARCH, GJR, and EGARCH. The study also compares the performance of these models and tries to determine the best model. To forecast the volatility of stock market return, the study examines the null hypothesis of homoscedastic normal process using the Engle (1982) test. This test is important also to measure the validity of using GARCH family to forecast the volatility of the return of the Egyptian stock market.

3. The Importance of the Study

This study is important to academics, policy makers, and economic agents because of several reasons: first, it is very important to the policy makers to preparing the developing and investment plans. Second, the forecasting of volatility is important for the fund manger how selecting the optimal portfolio depending on the return and the risk which measured by the volatility. Third, the test of the null hypothesis of homoscedastic normal process helps the policy maker, fund manger, and academic to choose the optimal model to predicting of financial market return volatility. Finally, the prediction of volatility of market return is very important to academics whose study in the field of finance and economic.

4. Literature Review

In 1952 Markowitz (1952) found out the idea of diversification to select the optimal portfolio. From this point Markowitz built the Modern Portfolio Theory. The portfolio problem is maximizing the expected return at a certain level of risk. Markowitz Portfolio Theory uses the standard deviation to measure the risk of stock return, so it uses equally weighted data. Therefore, it ignores the dynamic structure of the market.

Several recent models solved the problem of the dynamic nature of the market which called linear and Non-linear Volatility Models. The most popular non-linear financial models are the Autoregressive Conditional Heteroscedastic (ARCH) models or Generalized Autoregressive Conditional Heteroscedastic (GARCH) models, and Switching models.

Modeling and forecasting stock market volatility has been the focus of several empirical and theoretical studies over the past decade. The next section will discuss various volatility models that have been found to be useful for modeling financial data.

4.1. Exponentially Weighted Moving Average (EWMA)

Markowitz Portfolio Theory uses equally weighted data to calculate the standard deviation to measure the risk of the market return. Unfortunately, the equally weighted data don't reflect the current state of the market, because the current observation takes the same importance of the previous observations. RiskMetrics™ (1996) solved this problem by developing a model which estimates the conditional variance and covariance based on the Exponentially Weighted Moving Average (EWMA) model. EWMA technique is used for measuring volatility by weighting recent observation more heavily than the distant ones.

J.P Morgan RiskMetrics™ (1996) uses the following equation to calculate (EWMA):

$$\sigma = \sqrt{(1-\lambda)\sum_{t=1}^{T}\lambda^{t-1}(R_t - \overline{R})^2} \tag{1}$$

The parameter λ $(0 \prec \lambda \prec 1)$ used as decay factor. To choose its decay factor RiskMetrics processed 480 time series to produce the decay factor (λ) which minimize the root mean squared error (RMSE) of the variance of forecast.

In addition to this, to determine the time horizon taken in to consideration by EWMA, RiskMetrics™ (1996) uses the following formula:

$$T = \frac{In\alpha}{In\lambda} \tag{2}$$

Where α represent the tolerance level and thus $(1-\lambda)$ is the confidence level.

4.2. Autoregressive Conditional Heteroscedastic (ARHC) Models

The Autoregressive Conditional Heteroscedastic (ARHC) is one of the most important econometric models which are used widely in finance. ARHC (q) model was proposed by Engel (1982). The main assumption of ARHC is that the variance of the errors is not constant which known as heteroscedasticity, a full model of ARHC is given as following:

$$y_t = \phi_1 + \sum_{j=2}^{p} \phi_j x_{jt} + u_t \quad u_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j u_{t-1}^2 \tag{3}$$

If we look to the second equation, we will know that the "the autocorrelation" in volatility is modeled by allowing the conditional variance of the error term σ_t^2 to take a function of the previous value of the squared error, so the ARCH is one of the best solutions to deal with the problem of "Volatility Clustering" (Brooks,2008).

4.3. Generalized Autoregressive Conditional Heteroscedastic (GARHC) Models

The GARCH (p,q) Model was developed by Bollerslev (1986). According to GARCH Model, the conditional variance allowed to be depended upon the previous own lags as well as the squared residuals. The GARCH Model defined as:

$$y_t = \phi_1 + \sum_{j=2}^{p} \phi_j x_{jt} + u_t \quad u_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{4}$$

4.4. Asymmetric GARCH Models

One of the primary restrictions of GARCH Models is that they enforce a symmetric response of volatility to positive and negative shocks. This arises because of the conditional variance in the GARCH Model which takes a function of the squared lagged error (Brooks, 2008). There are several asymmetric models which solve the previous problem. Two popular asymmetric models are explained below:

4.4.1. The GJR model (Glosten et al, 1993)

The Glosten, Jagannathan, and Runkle (GJR) is a simple extension of GARCH Model with an additional term added to capture possible asymmetric. The advantage of GJR model is that it allows positive and negative innovations (good and bad news) to have different impacts on conditional variance. The negative innovations have higher impact on conditional variance than the positive innovations have.

In fact the distribution of stock return can be skewed. For example, skewed to left (there are more negative than positive outlying observation), the symmetric GARCH model cannot cope with such skewness. The GJR takes account of skewed distribution (Franses and Dijk, 1996). The GJR (1, 1) Model is given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \quad (5)$$

Where: $I_{t-1} = 1$ if $u_{t-1} \prec 0$

$I_{t-1} = 0$ Otherwise

For the case of $p = 1, u \succ 1$, $\alpha_1 + \gamma_1 \succ 0$, $\beta_1 \geq 0$ are sufficient conditions to ensure a strictly positive conditional variance. In the case of negative innovations, I_{t-1} takes a value of 1, the impact of negative innovations on σ_t^2 is $((\alpha + \gamma)u_{t-1}^2$. On the other side, on the case of positive shocks, I_{t-1} takes a value of zero, the impact of positive shocks on σ_t^2 is αu_{t-1}^2.

4.4.2. The EGARCH Model

The exponential GARCH Model was developed by Nelson (1991). EGARCH Model has various formulas. One of them is given by:

$$In(\sigma_t^2) = \omega + \beta In(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]. \quad (6)$$

There is no need to impose non-negative constraints on the model parameters, asymmetric are allowed for under the EGARCH formulation, since if the relationship between volatility and returns is negative, γ , will be negative (for more details, see Brooks, 2008).

4.5. The Empirical Studies of Forecasting Volatility

Many empirical studies investigated forecasting stock return volatility process. One of the most important studies in this field the study of Akgiray (1989), he used the daily stock return data in the USA market to evaluate four alternative stock return predicting models. These models are: Historical Average model, EWMA model, ARCH model, and GARCH model. He studied also the stock return distribution properties. His results agreed with the results of Fama (1965) that large price changes are followed by large changes, and small price changes are followed by small changes. The results showed also that the conditional heteroscedastic models (ARCH, GARCH models) beat the other models (Historical Average, EWMA). On the other hand, some studies interested with the parametric and non-parametric models of forecasting stock return volatility. For example, Pagan and Schwert (1990) compared several statistical models for monthly stock return volatility. They compared the non-parametric models of conditional volatility with the parametric models of conditional volatility (GARCH, EGARCH, and Hamilton (1989)) and argued that the non-parametric models give a better explanation of the squared returns than any parametric models. Also, they argued that both Hamilton's and GARCH give weak explanation of the data and on the other side, EGARCH model come closed to the explanatory power of the non-parametric models.

To evaluate GARCH model, Franses and Dijk (1996) investigated the performance of the GARCH model and two of its non-linear modifications (Quadratic GARCH (Engle and Ng, 1993) and Glosten, Jagannathan and Runkle (1992)) models. They concluded that the QGARCH model is the best model, and that the GJR model is not a useful tool for forecasting. The results showed that the GARCH (1, 1) model parameters α and β are usually significant at the 5% level, and hence that the constant variance model can be rejected. Another important study is the study of Jun ya (1999) which evaluated the performance of nine forecasting volatility models in the New Zealand Stock Market. These models are: The Random Walk model, Historical Average model, Moving Average model, Simple Regression model, Exponential Smoothing model, Exponential Moving Average model, ARCH model, GARCH model, and SV model.

5. Data and Methodology

5.1. Data

Daily return of three Egyptian stock market indices was calculated. These indices are: 1) EGX30, 2) CIBC100, and 3) Hermes Financial Index (HFI). EGX30 index includes the top 30 companies in term of liquidity and activity. EGX30 index weighted by market by market capitalization and adjusted by free float (see: www.egyptse.com). The CIBC 100 Index is an equally-weighted index comprising the most active 100 stocks traded on Egyptian Exchange, exhibiting the largest trading value in the year and taking into account traded volumes and recent public listings. Finally, The Hermes Financial Index (HFI) tracks the movement of the most active Egyptian stocks traded on the Egyptian Exchange. Criteria for inclusion in the index are average daily value traded, average daily number of transactions, total number of days traded during a calendar quarter and market capitalization. Figures (1), (2), (3) show the daily return of EGX30, CIBC100, and HFI indices on the period from 1-1-1998 until 31-12-2009.

Figure 1: Daily return of EGX30 index on the period from 1 - 1 - 1998 until 31 -12- 2009"

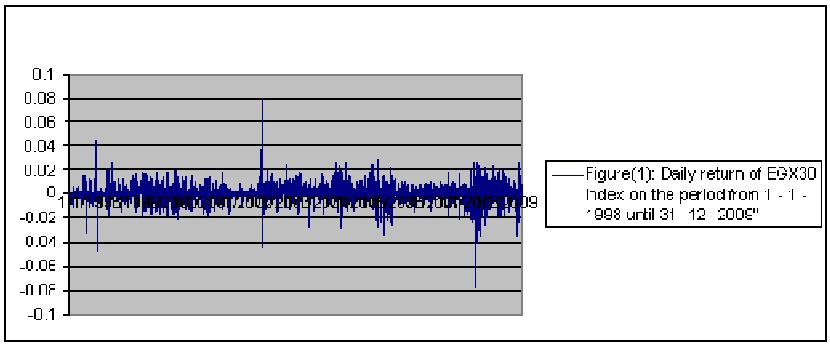


Figure 2: Daily return of CIBC100 index on the period from 1-1-1998 until 31-12-2009

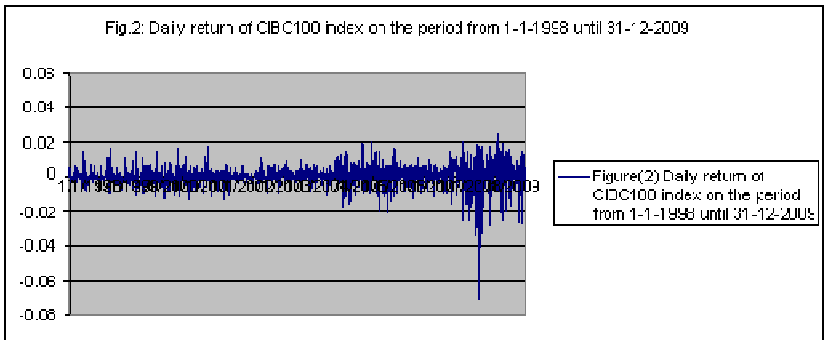
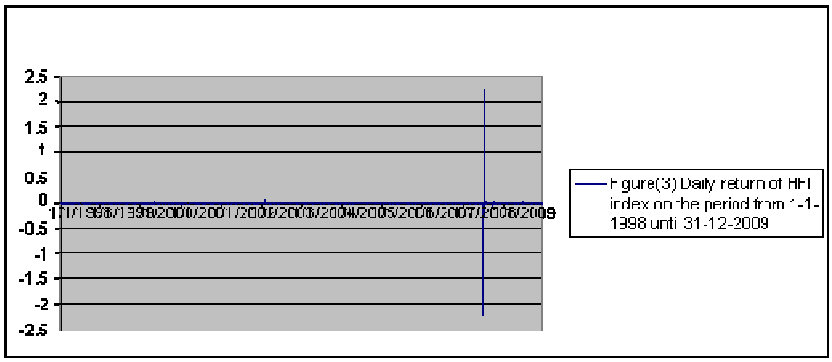


Figure 3: Daily return of HFI index on the period from 1-1-1998 until 31-12-2009



This market indices time series cover the period from January 1998 to December 2009. So, they contain approximately 2956 daily observation, this period is the in-sample period. The daily return defined as the natural logarithm of price relatives:

$$R_t = \log \frac{P_t}{P_{t-1}}$$

Where P_t is the daily capital indexc.

On the other hand, we used the next 30 days after the in-sample period to be our out-of-sample period.

5.2. Descriptive Statistics

In total we have three stock indices. Table (1) shows the descriptive statistics of the three indices. As we can see, the descriptive statistics of the EGX30 index return shows that, the sample maximum is 0.079776, the sample minimum is -0.078137, and the sample Kurtosis is 12.381120 (fat tail, whose source may be the volatility clustering). The descriptive statistics of the other indices show also that it have fat tail[1]. But until new we have no evidence of the volatility clustering in the indices time series. Figures (1), (2), (3) show the daily return of both EGX30, CIBC100, and HFI indices on the in-sample period (1-1-1998 until 31-12-2009).

Table 1: The descriptive statistics of the three indices.

	Egx30	**CIBC100**	**HFI**
Mean	0.000268	0.000174	0.000217
Median	0.000156	0.000179	0.000252
Maximum	0.079776	0.048885	2.225640
Minimum	-0.078137	-0.070928	-2.222612
Std. Dev.	0.007926	0.005958	0.056858
Skewness	-0.199471	-0.994868	0.066462
Kurtosis	12.381120	17.429570	1507.156000
Jarque-Bera	10855.27	26512.61	293000000.00
Probability	0.000000	0.000000	0.000000
Sum	0.792897	0.522575	0.674085
Sum Sq. Dev.	0.185597	0.106433	10.047680
Observations	2955	2999	3109

5.3. ARCH Effect Test

Before estimating a GARCH models, it is important to first to run the Engle (1982) test for the ARCH effect (Brooks, 2008) to make sure that this model is appropriate for the data. This test called also Lagrange Multiplier (LM). This test was modified by the observation that in many financial time series, the magnitude of residuals appeared to be related to the magnitude of recent residuals. After run the regression of the time series of the market index, the residuals squares regressed on (q) own lags:

$$\hat{u}_t^2 = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2 + \gamma_2 \hat{u}_{t-2}^2 + ... + \gamma_q \hat{u}_{t-q}^2 + \upsilon_t \quad (7)$$

Where υ_t is an error term.

The test statistic is distributed as a $\chi^2(q)$.

Where:

$H_0 : \gamma_1 = 0$ and $\gamma_2 = 0$ and $\gamma_3 = 0$ and … $\gamma_q = 0$

$H_1 : \gamma_1 \neq 0$ and $\gamma_2 \neq 0$ and $\gamma_3 \neq 0$ and … $\gamma_q \neq 0$

[1] This agrees with Mandelbrot (1963).

The Lagrange Multiplier (LM) test statistic in the table (2) indicates the presence of significant ARCH effect in both the EGX30 and CIBC100 index. The null hypothesis of homoscedastic normal process was rejected. On the other hand, the LM test statistic shows no significant ARCH effect in HFI index. So, in this study we will consider both the EGX30 index and CIBC100 index, and exclude HFI index.

Table 2: Test statistic and probability value of the LM test of the three indices.

	Egx30		CIBC100		HFI	
	Coefficient	**p -value**	**Coefficient**	**p -value**	**Coefficient**	**p -value**
γ_0	0.000038	0.000000	0.000021	0.000000	0.001877	0.266200
γ_1	0.269767	0.000000	0.165712	0.000000	0.034686	0.060000
γ_2	0.028716	0.132300	0.047092	0.013200	0.025288	0.170400
γ_3	0.022518	0.237900	0.047410	0.012500	0.013500	0.464400
γ_4	0.033952	0.075100	0.049414	0.009300	0.007577	0.681200
γ_5	0.027742	0.132200	0.081114	0.000000	0.004880	0.791200
LM statistic	254.256600	0.000000	155.074800	0.000000	6.662530	0.246972

5.4. Forecasts of Volatility

The previous section suggested the presence of ARCH effect in EGX30 and CIBC100 index. So, we can conclude that the GARCH model family is appropriate for the data.

In this study we will investigate the performance of five volatility predicting models. These models are; Exponentially Weighted Moving Average model (EWMA), The Autoregressive Conditional Heteroscedastic (ARHC) model, The General Autoregressive Conditional Heteroscedastic (GARHC), GJR model, and EGARCH model. The previous models has discussed previously.

5.4.1. Deciding the Appropriate Model Order Using Information Criteria

The selection of GARCH orders p, and q is an important issue. Since GARCH models could be treated as ARMA models (Brooks, 2008), traditional information criteria models could be used for selecting models. The most three popular information criteria models are: Akaike's (1974) Information Criterion (AIC), Schwarz's (1978) Bayesian Information Criteria (SBIC), and Hannan-Quinn's (1979) Criteria (HQIC).

Since no criterion is definitely superior to others (Brooks, 2008), we will consider only AIC criterion to determine the GARCH models orders. The AIC criterion defined as:

$$AIC_{\ell} = -2\ell / T + \frac{2k}{T} \tag{8}$$

Where:

$$\ell = -\frac{T}{2}(1 + In(2\pi) + In(\hat{u}'\hat{u} / T)) \tag{9}$$

Where, u is the regression's disturbance, k is the number of parameters, and T is the sample size.

The GARCH orders which minimize the value of AIC Information Criteria should be chosen.

Tables (3), (4) give model selection criteria for different combinations of GARCH (p,q) for daily return of the EGX30 and CIBC100 indices respectively. In table (3) For ARCH (p) models, the optimal ARCH orders, witch minimizes the value of AIC, is (3). So ARCH (3) was chosen.

On the other hand, for AR(1)-GARCH (p,q) models, the AR(1) - GARCH (3,3) minimizes the value of AIC. So AR(1)-GARCH (3, 3) was chosen.

Table 3: AIC model selection criteria for estimated GARCH for daily return of EGX30 index.

p/q	0	1	2	3
1	-6.9819	-7.1276	**-7.1304**	-7.1383
2	-7.0133	-7.13307	-7.1325	-7.13956
3	**-7.0188**	-7.13248	-7.1323	**-7.1444**

Table 4: AIC model selection criteria for estimated GARCH for daily return of CIBC index

p/q	0	1	2	3
1	-7.573398	**-7.911585**	-7.911149	-7.911737
2	-7.682273	-7.910995	**-7.918406**	-7.911968
3	**-7.746541**	-7.910824	-7.911966	-7.915566

If we look to table (4), we will conclude the following results: For CIBC100 index, the optimal ARCH orders, witch minimizes the value of AIC, is (3). So AR(1)-ARCH (3) was chosen. For GARCH (p,q) models, the AR(1)-GARCH (2,2) minimizes the value of AIC. So AR(1)-GARCH (2, 2) was chosen.

5.4.2. Estimating of Volatility Models

Since HFI index has no significant ARCH effect, we considered only the daily returns of EGX30 and CIBC100 indices to estimate the predicted market return volatility. To forecast the variance of return, five volatility predicting models were used. These models are: EWMA model, ARCH model, GARCH model, GJR model, and EGARCH model.

5.4.2.1. EWMA Forecast

To estimate EWMA parameters, we use 48 daily returns as an in-sample period. The in-sample day's number was calculated using equation (2). 30 observations of daily volatility were forecasted as an out-of-sample period. Table (5) includes the results of EWMA process to daily return of both EGX30 and CIBC100 indices.

Table 5: The forecasted out-of-sample values of both EGX30 and CIBC100 indices daily volatility using EWMA model

days	EGX30 Index σ_{EWMA}	CIBC100 Index σ_{EWMA}
1	0.008535	0.005323
2	0.008312	0.006692
3	0.008141	0.006827
4	0.008122	0.007047
5	0.007861	0.006871
6	0.007640	0.007034
7	0.007348	0.006717
8	0.007226	0.006542
9	0.007021	0.00617
10	0.006962	0.00587
11	0.006731	0.00572
12	0.006498	0.005579
13	0.006539	0.005713
14	0.006353	0.005491
15	0.006199	0.005388
16	0.006817	0.006133
17	0.006758	0.006158
18	0.006561	0.006219
19	0.006569	0.006306

Table 5: The forecasted out-of-sample values of both EGX30 and CIBC100 indices daily volatility using EWMA model - continued

20	0.006347	0.006417
21	0.006145	0.006295
22	0.006196	0.006222
23	0.006166	0.006041
24	0.006079	0.006071
25	0.006433	0.005923
26	0.006397	0.005703
27	0.006209	0.005529
28	0.005999	0.005355
29	0.005798	0.005183
30	0.005708	0.005311

5.4.2.2. ARCH Forecast

Tables (6), (7) show the results of ARCH process of EGX30 and CIBC100 indices. The results show that all of the parameters α_i are statistically significant for both EGX30 and CIBC100 indices. The sum of the ARCH parameters $(\alpha_1 + \alpha_2 + ... + \alpha_p)$ is smaller than unity. This indicates that the fitted model is second-order stationary (Akgiry, 1989).

Table 6: AR(1) - ARCH (3) Model Estimates For EGX30 index

Parameter	Coefficient	Std. Error	z-Statistic	P-value
Mean Equation				
ϕ_0	0.000228	0.000144	1.581006	0.1139
ϕ_1	0.205075	0.021232	9.65863	0
Variance Equation				
α_0	0.000033	5.49E-07	59.35998	0
α_1	0.229398	2.13E-02	10.74551	0
α_2	0.159234	0.025879	6.152936	0
α_3	0.086191	0.020585	4.186978	0
$\alpha_1 + \alpha_2 + \alpha_3$	0.474823			
Log likelihood	10376.31	AIC		-7.01882
F-statistic	18.29833	Prob(F-statistic)		0

Table 7: AR(1) - ARCH (3) Model Estimates For CIBC100 index

Parameter	Coefficient	Std. Error	z-Statistic	P-value
Mean Equation				
ϕ_0	0.000132	0.000109	1.220577	0.2222
ϕ_1	0.307681	0.013135	23.42535	0
Variance Equation				
α_0	0.000010	2.62E-07	39.60078	0
α_1	0.279957	0.019842	14.10949	0
α_2	0.323033	0.012963	24.91965	0
α_3	0.264302	0.019582	13.49746	0
$\alpha_1 + \alpha_2 + \alpha_3$	0.867292			
Log likelihood	10998.34	AIC		-7.74654
F-statistic	6.395204	Prob(F-statistic)		0.000006

In mean equation ϕ_0 denotes the constant parameter, ϕ_1 denotes the Autoregressive (AR) parameters for the daily return. We can see that ϕ_0 is insignificant for both EGX30 and CIBC100 indices, on the other hand ϕ_1 is significant for both EGX30 and CIBC100 indices.

5.4.2.3. GARCH Forecast

Since the GARCH model is non-linear model, ordinary least squares (OLS) cannot be used to estimate it. To estimate any type of GARCH models, another technique known as *Maximum Likelihood* is used (Brooks, 2008).

To specify the Log- Likelihood Function (LLF), the next formula will be used:

$$L = -\frac{T}{2} Log(2\pi) - \frac{1}{2}\sum_{t=1}^{T} Log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T} (y_t - \mu - \phi y_{t-1})^2 / \sigma_t^2 \quad (10)$$

To select the optimal GARCH model's orders, AIC criteria were used as discussed above. The minimum AIC value for EGX30 was for AR(1)-GARCH (3, 3). But if we look at table (8), we will see that in AR(1)-GARCH (3, 3) non-negativity restriction on coefficients is violated. For example, in AR(1)-GARCH (3, 3) β_1 coefficient of GARCH is -0.111453, so we selected the second lowest value of AIC. We can see also that AR(1)-GARCH (2, 3), GARCH AR(1)- (1, 3), AR(1)-GARCH (2, 1), AR(1)-GARCH (3, 1), AR(1)-GARCH (2, 2), and GARCH AR(1)- (3, 2) violated the constraint of non-negativity. So, we selected AR(1)-GARCH (1, 2) which have the lowest AIC value and don't violate the constraint of non-negativity. So we selected the same orders for the other GARCH models as follows: AR(1)-GJR (1, 2), AR(1)-EGARCH (1, 2).

Table (9) includes the results of different combinations of p and q for GARCH processes of CIBC100 index daily return series. Likewise, we can see that non-negativity restriction on coefficients is violated. In AR(1)-GARCH (2, 2) the coefficient α_2 takes the value of -0.185401 which violates the constraint of non-negativity. We can see also that AR(1)-GARCH (3, 3), AR(1)-GARCH (2, 3), AR(1)-GARCH (3, 2), and AR(1)-GARCH (1, 3) violated the constraint of non-negativity. So, we selected AR(1)-GARCH (1, 1) which have the lowest AIC value and don't violate the constraint of non-negativity. We selected the same orders for the other GARCH models as follows: AR(1)-GJR (1, 1), AR(1)-EGARCH (1, 1).

Table (10) includes the full results of AR(1)-GARCH (1, 2) process of EGX30 index. The results show that the parameters estimates of AR(1)-GARCH (1, 2) are all statistically significant, both $\alpha_i's$ and $\beta_i's$ are statistically significant. The stationary of time series of data is an important thing. We can conclude this stationary through the stationary condition that the summation of $\alpha + \beta$ close to unity. From table (10), we can see that $\alpha_1 + \beta_1 + \beta_2 = 0.988788$ which is close to and smaller than unity. So we can conclude the second order stationary of the time series. We can conclude that the large changes in returns tend to be followed by larger changes and small changes tend to be followed by smaller changes (Brooks, 2008), (Kovacic, 2008).

If we look at the mean equation section we will see that ϕ_1 AR (1) is significant, which capture the linear process in the return series of EGX300 index.

Table 8: GARCH process estimates for variance parameters of EGX30 index daily return using different combinations of p and q.

Parameter	GARCH(3,3)	GARCH(2,3)	GARCH(1,3)	GARCH(2,1)	GARCH(3,1)	GARCH(2,2)	GARCH(3,2)	GARCH(1,2)
α_0	0.000002 (0.000000)	0.000001 (0.000000)	0.000002 (0.000000)	0.000001 (0.000000)	8.40E-07 (0.000000)	0.000001 (0.002100)	0.000001 (0.000900)	1.45E-06 (0.000000)
α_1	0.159490 (0.000000)	0.133891 (0.000000)	0.236857 (0.000000)	0.204392 (0.000000)	0.202551 (0.000000)	0.203433 (0.000000)	0.201209 (0.000000)	0.164734 (0.000000)
α_2	0.120616 (0.000000)	-0.004276 (0.572800)		-0.105117 (0.000000)	-0.092917 (0.002500)	-0.112830 (0.000000)	0.036825 (0.572900)	
α_3	0.029974 (0.015800)				-0.013119 (0.526600)		-0.088537 (0.004100)	
β_1	-0.111453 (0.000000)	1.425958 (0.000000)	0.147559 (0.000000)	0.893545 (0.000000)	0.896400 (0.000000)	0.966687 (0.000000)	0.327455 (0.334700)	0.465656 (0.000200)
β_2	0.009680 (0.448400)	-1.286878 (0.000000)	-0.031198 (0.038600)			-0.063913 (0.704000)	0.512710 (0.087300)	0.358398 (0.002000)
β_3	0.788143 (0.000000)	0.721017 (0.000000)	0.635450 (0.000000)					
$\alpha+\beta$	0.996450	0.989712	0.988668	0.992820	0.992915	0.993377	0.989662	0.988788
Log likelihood AIC	10564.870 -7.144414	10556.700 -7.139562	10553.770 -7.138256	10545.110 -7.133067	10545.250 -7.132484	10545.200 -7.132451	10545.950 -7.132286	10541.21 -7.13043

Note: values between parentheses show probability values.

Table 9: GARCH process estimates for variance parameters of CIBC100 index daily return using different combinations of p and q.

	GARCH(2,2)		GARCH(3,3)		GARCH(2,3)		GARCH(3,2)		GARCH(1,3)		GARCH(1,1)	
Parameter	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
α_0	0.000000	0.002100	0.000002	0.000000	0.000001	0.000000	0.000001	0.000000	0.000001	0.000000	0.000001	0.000000
α_1	0.189680	0.000000	0.167260	0.000000	0.195656	0.000000	0.195364	0.000000	0.203530	0.000000	0.179518	0.000000
α_2	-0.185401	0.000000	-0.025984	0.093800	0.107713	0.002700	0.130088	0.000000				
α_3			0.211596	0.000000			0.014721	0.430700				
β_1	1.729542	0.000000	0.852921	0.000000	0.045494	0.757300	-0.064542	0.190000	0.450450	0.000000	0.811409	0.000000
β_2	-0.733868	0.000000	-0.800610	0.000000	0.714412	0.000000	0.706308	0.000000	0.548239	0.000000		
β_3			0.578371	0.000000	-0.079388	0.426600			-0.211908	0.000600		
$\alpha+\beta$	0.999953		0.983554		0.983887		0.981939		0.990311		0.990927	
Log likelihood AIC	11243.220 -7.918406		11241.190 -7.915566		11235.080 -7.911968		11235.080 -7.911966		11233.750 -7.911737		11231.540 -7.911585	

On the other hand, table (11) shows the full results of AR(1)-GARCH (1, 1) process of CIBC100 index. The results in this table do not different more from that in table (10). In the mean equation, the Autoregressive coefficient parameter ϕ_1 is highly significant. All the parameters of the volatility equation are highly significant, and its summation closes to and smaller than the unity. The sum of $\alpha_1 + \beta_1 + \beta_2$ equals 0.990927

Table 10: AR(1) - GARCH (1 , 2) Model Estimates For EGX30 index

	Coefficient	Std. Error	z-Statistic	P-value.
		Mean Equation		
ϕ_0	0.000211	0.000134	1.58047	0.114
ϕ_1	0.205936	0.018596	11.07397	0
		Variance Equation		
α_0	1.45E-06	2.04E-07	7.085439	0
α_1	0.164734	0.013889	11.86106	0
β_1	0.465656	0.126805	3.672223	0.0002
β_2	0.358398	0.115747	3.09638	0.002
$\alpha + \beta$	0.988788			
Log likelihood	10541.21	AIC		-7.13043
F-statistic	18.25553	Prob(F-statistic)		0

Table 11: AR(1) - GARCH (1 , 1) Model Estimates For CIBC100 index

	Coefficient	Std. Error	z-Statistic	P-value.
		Mean Equation		
ϕ_0	0.000155	9.59E-05	1.617313	0.1058
ϕ_1	0.247095	0.019511	12.66442	0
		Variance Equation		
α_0	7.35E-07	6.18E-08	11.89859	0
α_1	0.179518	0.009417	19.06272	0
β_1	0.811409	0.007116	114.0305	0
$\alpha + \beta$	0.990927			
Log likelihood	11231.54	AIC		-7.91159
F-statistic	17.38188	Prob(F-statistic)		0

5.4.2.4. GJR Forecast

As we showed previously, GJR is an asymmetric model which allows to the positive and negative innovations to have different effects on the conditional variance σ^2.

Table (12), (13) show the results of GJR processes for EGX30, CIBC100 indices. If we look at table (12) we can see that both α_0 (constant), α_1 (ARCH effect), and β (GARCH effect) are highly significant. The sum of estimated α_1 and βs is less than and close to unity. On the other hand, the coefficient γ which reflects leverage effect is insignificant and has negative sign. This implies that the positive innovations have higher impact on the conditional variance σ^2 than that the negative innovations have. This indicates the absence of leverage effect in the daily return of EGX30 index.

Table 12: AR(1) -a GJR (1 , 2) Model Estimates For EGX30 index

	Coefficient	Std. Error	z-Statistic	P-value.
Mean Equation				
ϕ_0	0.000234	0.000138	1.688958	0.091200
ϕ_1	0.205567	0.018604	11.049720	0.000000
Variance Equation				
α_0	0.000001	0.000000	6.871791	0.000000
α_1	0.172942	0.015430	11.207810	0.000000
γ	-0.018584	0.014991	-1.239725	0.215100
β_1	0.463331	0.127846	3.624143	0.000300
β_2	0.362404	0.116796	3.102891	0.001900
$\alpha+\beta$	0.998677			
Log likelihood	10541.57	AIC		-7.129996
F-statistic	15.230730	Prob(F-statistic)		0.000000

Table 13: AR(1) - GJR (1 , 1) Model Estimates For CIBC100 index

	Coefficient	Std. Error	z-Statistic	P-value
Mean Equation				
ϕ_0	0.000157	0.000101	1.557599	0.1193
ϕ_1	0.247127	0.019517	12.66182	0
Variance Equation				
α_0	0.000001	0.000000	11.52076	0
α_1	0.180188	0.010493	17.17157	0
γ	-0.001560	0.016943	-0.092086	0.9266
β_1	0.811524	0.007916	102.5154	0
$\alpha+\beta$	0.991712			
Log likelihood	11231.54	AIC		-7.91088
F-statistic	13.89901	Prob(F-statistic)		0

Table (13) displays the results of GJR processes for CIBC100 index. The results in this table are similar to the results in table (12). All the parameters of ARCH, GARCH effects are significant. The coefficient γ is insignificant; so we conclude also the absence of the leverage effect in the daily return of CIBC index. Tables (12), (13) show also the coefficient ϕ_1 is significant.

5.4.2.5. EGARCH Forecast

Table (14) shows the results of EGARCH process. We can see that the coefficient γ is positive and significant at 5%, and 10% confidence levels; this implies the absence of leverage effect in the daily return of EGX30 index. So, we expect that the positive shocks have a higher impact on the conditional variance than that the negative shocks have.

Table 14: AR(1) - EGARCH (1 , 2) Model Estimates For EGX30 index

	Coefficient	Std. Error	z-Statistic	Prob.
Mean Equation				
ϕ_0	0.000283	0.000126	2.240063	0.0251
ϕ_1	0.208322	0.017476	11.92048	0

Table 14: AR(1) - EGARCH (1 , 2) Model Estimates For EGX30 index - continued

Variance Equation				
α_0	-0.6029	0.039647	-15.2068	0
α_1	0.305421	0.01723	17.72606	0
γ	0.015854	0.007699	2.059052	0.0395
β_1	0.73524	0.058849	12.49358	0
β_2	0.226772	0.057978	3.911365	0.0001
Log likelihood	10535.19	AIC		-7.125677
F-statistic	15.14785	Prob(F-statistic)		0

Table 15: AR(1) - EGARCH (1 , 1) Model Estimates For CIBC100 index

	Coefficient	**Std. Error**	**z-Statistic**	**Prob.**
Mean Equation				
ϕ_0	0.000239	9.48E-05	2.522964	0.0116
ϕ_1	0.243348	0.016622	14.64027	0
Variance Equation				
α_0	-0.581486	0.029283	-19.85767	0
α_1	0.322294	0.012661	25.45576	0
γ	0.00036	0.008674	0.041525	0.9669
β_1	0.967638	0.002341	413.256	0
Log likelihood	11217.29	AIC		-7.900839
F-statistic	14.22466	Prob(F-statistic)		0

We can see also that all the parameters of ARCH, GARCH effects in table (14) are significant.

The results in table (15) are similar to these in table (14); all the parameters of ARCH, GARCH effects are significant. The coefficient γ is positive but insignificant. So we conclude also the absence of leverage effect in the daily return of CIBC100 index.

5.5. Evaluating Forecasting Models

In this section we will evaluate the performance of the five models of predicting volatility which we used previously. To evaluate the goodness-of-fit of the five models, three evaluating measures used to evaluate out-of-sample forecasting accuracy. These models are: the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE), and Mean Absolute Percent Error (MAPE). We can define these models as:

$$RMSE = \sqrt{\frac{1}{T-(T_1-1)}\sum_{t=T_1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2} \tag{11}$$

Where $\hat{\sigma}_t^2$ is the one-step-ahead forecast volatility, σ_t^2 is the actual volatility, and T is the total size (in-sample + out-of-sample), and T_1 is the first out-of-sample forecast observation.

The MAE is defined by:

$$MAE = \frac{1}{T-(T_1-1)}\sum_{t=T_1}^{T}|\hat{\sigma}_t^2 - \sigma_t^2| \tag{12}$$

The MAPE is defined by:

$$MAPE = \frac{100}{T-(T_1-1)}\sum_{t=T_1}^{T}\left|\frac{\hat{\sigma}_t^2 - \sigma_t^2}{\sigma_t^2}\right| \tag{13}$$

The process of evaluating of the performance of the models of predicting volatility requires strongly calculating the true volatility. We will use the following model to calculate the "*true volatility*" ((Pagan and Schwert (1990)), (Day and Lewis (1992), and (Chong (1999)) :

$$v_t = (r_t - \bar{r})^2 \tag{14}$$

6. Results and Discussion

Tables (16) and (17) include the results of the evaluation of the out-of-sample volatility forecasts of both EGX30 and CIBC100 indices. In these tables the value and ranking of all five models were reported under the RMSN, MAE, and MAPE measures. From table (16) we see that both RMSE and MAE indicates that AR (1) - EGARCH (1, 2) Model provides the most accurate forecasts while AR(1) - GJR (1 , 2) Model ranks second, AR(1) - ARCH (3) Model ranks third, AR (1) - GARCH (1, 2) Model ranks fourth, and EWMA came in the last position. On the other hand, MAPE statistics gives reverse results; EWMA came on the first rank, AR(1) - GARCH (1 ,2) ranks second, AR(1) - ARCH (3) ranks third, AR(1) - GJR (1 ,2) ranks forth, and AR(1) - EGARCH (1 ,2) came in the last position.

From table (17), EWMA came in the first position according to the three evaluating measures, and AR (1) - EGARCH (1, 1) ranks second according to the three evaluating measures also. If we look to the other models, we will note that: according to RMSE and MAPE AR (1) - GJR (1, 1) ranks third, AR (1) - GARCH (1, 1) ranks fourth and AR (1) - ARCH (3) ranks fifth.

Finally, according to MAE, AR (1) - ARCH (1, 1) ranks third, AR (1) - GJR (1, 1) ranks fourth, and AR (1) - GARCH (3) ranks fifth.

Table 16: Evaluation of the out-of-sample volatility forecasts of EGX30 index

Model	RMSE		MAE		MAPE	
	value	rank	value	rank	value	rank
EWMA	0.005322	5	0.004138	5	60.937863	1
AR(1) - ARCH (3)	0.005207	3	0.004046	3	93.311850	3
AR(1) - GARCH (1 ,2)	0.005210	4	0.004048	4	92.377250	2
AR(1) - GJR (1 ,2)	0.005206	2	0.004044	2	93.617150	4
AR(1) - EGARCH (1 ,2)	0.005196	1	0.004035	1	96.257910	5

Table 17: Evaluation of the out-of-sample volatility forecasts of CIBC100 index

Model	RMSE		MAE		MAPE	
	value	rank	value	rank	value	rank
EWMA	0.004732	1	0.003688	1	60.845737	1
AR(1) - ARCH (3)	0.005439	5	0.004291	3	176.726100	5
AR(1) - GARCH (1 , 1)	0.005416	4	0.004295	5	157.299800	4
AR(1) - GJR (1 , 1)	0.005415	3	0.004294	4	157.262600	3
AR(1) - EGARCH (1 , 1)	0.005402	2	0.004277	2	156.725400	2

6.1. Diebold-Mariano (DM) Test

As we saw in the previous section, we can't test the significance of the equality of forecast accuracy using the usual statistical metrics (RMSE, MAE, and MAPE). In this section we will use Diebold-Mariano (1995) test to comparing the performance of the five volatility models. According to DM test, we will test the null hypothesis of equal forecast accuracy for two forecasts. Let $\{e_{it}\}_{t=T_1}^{T}$ and $\{e_{jt}\}_{t=T_1}^{T}$ denote forecast errors from two different volatility predicting models. The accuracy of each forecast is measured by particular loss function. $L(e_{it})$, $i = 1,2$ where $L(e_{it}) = (e_{it})^2$. The DM test is based on the loss differential:

$$d_t = L(e_{it}) - L(e_{jt}) \tag{15}$$

The null hypothesis of equal forecast accuracy for two forecasts is:

$H_o : E[d_t] = 0$

The DM test statistics is:

$$S = \frac{\bar{d}}{\sqrt{(aver(\bar{d}))}} \tag{16}$$

Where $\bar{d} = \frac{1}{T-(T_1-1)} \sum_{t=T_1}^{T} d_t$ (17)

Tables (18), (19) show the DM statistics values for the differences between errors of volatility forecasting models and error of GARCH model for EGX30 and CIBC100 indices respectively. We can see that $P-value$ of all DM statistics takes the value of one. Thus, we don't reject the hypothesis of equal forecast accuracy of the five volatility forecasting models.

Table 18: T-Test of " mean difference = 0 (vs > 0)" for the difference between errors of the volatility forecasting models and the error of GARCH model for EGX30 index

	GARCH		
	difference	**P-Value**	**T-Value**
EWMA	-0.00031000	1.00000000	-5.80000000
ARCH	-0.00000054	1.00000000	-11.78000000
GJR	-0.00000085	1.00000000	-12.99000000
EGARCH	-0.00000310	1.00000000	-13.47000000

Table 19: T-Test of " mean difference = 0 (vs > 0)" for the difference between errors of the volatility forecasting models and the error of GARCH model for CIBC100 index

	GARCH		
	difference	**P-Value**	**T-Value**
EWMA	-0.00042000	1.00000000	-7.67000000
ARCH	-0.00000570	1.00000000	-4.09000000
GJR	-0.00000008	1.00000000	-4.60000000
EGARCH	-0.00000310	1.00000000	-17.24000000

In summary, according to the statistical metrics, we see that EGARCH outperforms the other models in forecasting the Egyptian market indices; since it ranks first in RMSE and MAS which were used to evaluate the forecasts of EGX30 index, and ranks second for the three measures which were used to evaluate the forecasts of CIBC100 index. These results agree with (Choo Wei et al, 1999), (Kovacic, Z.J, 2008), (Pagan and Schwert, 1990), (Figleweski, and Hasbrouck, 1993), (Tsay, 1992). On the other hand this result disagrees with (Rashid al., 2008). Thus, we consider that these results are weak because the DM test didn't support these results.

7. Conclusion

In this paper we examined five models for forecasting volatility of the Egyptian stock market index. We used three market indices to reflect the Egyptian market index; these indices are: EGX30, CIBC100, and HFI. The in-sample period was from January 1998 to December 2009. To predict the volatility of these indices we used out-of-sample period 30 days.

The validity of the usage of GARCH family models on the market stock indices was examined using Engle (1982) test for the ARCH effect (LM test). As we saw, the null hypothesis of homoscedastic normal process was rejected for both EGX30 and CIBC100; so the usage of GARCH

family models was appropriate for these indices. The optimal orders (p and q) of GARCH models were determined using AIC criteria whose results showed that the optimal orders are ($p = 1$ and $q = 2$) for EGX30 index and ($p = 1$ and $q = 1$) for CIBC100 index.

After that, we estimated the volatility models parameters. And finally, we evaluated forecasting models using two algorithms; the usual statistical metrics (RMSN, MAE, and MAPE) and using Diebold and Mariano (DM) test statistics. According to the usual statistical metrics, EGARCH model beat the other volatility forecasting models for the Egyptian stock market. But on the other hand, DM statistic showed no significant differences between the forecasting volatility models performance.

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