International Research Journal of Finance and Economics
ISSN 1450-2887 Issue 165 January, 2018
http://www.internationalresearchjournaloffinanceandeconomics.com

The Variation of Fama-French Three-Factor Beta Risks by Interval Test in Taiwan Stock Market: Theory and Evidence

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Abstract

In this research the three-factor Fama-French regression model (1992, 1993, 1995) is investigated in Taiwan stock market, in which the factors include the market risk premium (MRP), small-minus-big risk premium (SMB) and high-minus-low risk premium (HML) associated with the regression parameters ($\beta_{MRP},\beta_{SMB},\beta_{HML}$). It is known that the MRP, SMB and HML can affect a stock portfolio's return. Based on the Fama-French model, six types of stock portfolios (namely, BH, BL, BM, SH, SL, and SM) are created according to company size (Small or Big) and the ratio of book-to-market equity (High, Medium or Low). Using the data from the Taiwan Economic Journal (TEJ), a traditional multiple regression equation is proposed, which can explain the returns of six types of portfolios (BH, BL, BM, SH, SL, and SM) with R-square ranging from 93% to 97% based on the three factors (MRP, SMB, and HML). This study further tests the equivalence of the beta risk parameters ($\beta_{MRP},\beta_{SMB},\beta_{HML}$) using two one-sided tests (TOST) for the six types of portfolios under each factor, and the economically meaningful equivalence margins of these risk parameters were empirically determined by the concept of risk and return. Compared with traditional point test, TOST for beta risk parameters provides more information for investors.

Keywords: Fama-French three-factor model, Traditional test, Risk and return, Two one-sided tests (TOST), Equivalence
JEL classification codes: G11

1. Introduction

1.1 The Interpretation of Risk and Return in the Fama-French Model

The Fama-French three-factor model (1992, 1993, 1995) is a method used by finance professionals to explain the relation between risk and return of an equity portfolio. The three-factor model assesses portfolios based on three distinct risk factors found in the equity market to assist in analyzing portfolio's returns. Before the three-factor model, there was a well-known Capital Asset Pricing Model (CAPM), which used a single market risk factor to explain a portfolio's return.

CAPM takes into account the expected risk of the stock market, known as "betasystematic risk",and it compares the risk of an investment with the market. This is the reason; investors should be compensated by the market risk premium (MRP) when investing in a portfolio with higher systematic risk.All diversified stock portfolios have their own beta values. After testing CAPM on a great number of portfolios, it is well known that, on average, a portfolio's beta explains about 70% of all of its actual returns. In the Fama-French Three-Factor Model, MRP is still the most important risk factor, accounting for 70% of a typical diversified portfolio's return. However, the size of a company and its book-to-market value in a portfolio also has significant effects.

Fama and French (1993) tested thousands of random stock portfolios against their model, and found that a combination of market risk (MRP), size (SMB), and value (HML) explained about 95% of variations of a diversified portfolio's returns in US stock market. This is evidence that size (SMB) and book-to-market equity (HML) are indeed proxy for sensitivity to common risk factors in stock portfolio returns. The Fama-French Three-Factor Model thus has a far more explanatory power than using the traditional CAPM.

Fama-French (1995) created six types of portfolios (BH, BL, BM, SH, SL, and SM) by classifying the two groups of market equities (ME) and the three groups of book to market equity (BE/ME) ratios into six portfolios (2x3=6) where the return in each type of portfolio is market value-weighted. The reason for the classification of the six types of portfolios is that over thousands of trials Fama and French (1995) found that the two by three classifications can give the highest explanatory power (R-square) and furthermore, the SMB and HML are the best explanatory factors in addition to MRP for the variation of a stock portfolio's return. The explanatory variables used in this study are the excess return on the value-weighted market portfolio, MRP[1](the excess market returns), SMB (small minus big) and HML (high minus low); the response variables are the returns of the six types of portfolios.

SMB is the difference between the simple average of the returns on the three small-stock portfolios (SH, SL, and SM) and the simple average of the returns on the three big-stock portfolios (BH, BL, and BM). Thus, SMB is the difference between the returns on small- and big-stock portfolios. HML is the difference between the simple average of the returns on the two high-BE/ME portfolios (BH and SH) and the simple average of the returns on the two low- BE/ME portfolios (BL and SL). Thus, HML is the difference between the returns on high- and low-stock portfolios. Therefore, as in Fama-French (1993, 1995), the three risk factors (MRP, SMB, and HML) through the regression parameters capture most of the strong spread in the average returns on the six types of portfolios.

Following Fama and French (1992, 1993, 1995), the current work also uses six types of portfolios and thus investigate their rates of returns based on regressions on the three factors of MRP, SMB, and HML associated with three types of beta risks ($\beta_{MRP,}\beta_{SMB,}\beta_{HML}$)in Taiwan stock market.

1.2 The Purpose and Main Contribution of this Study

In this study, the book equity, market equity (the number of outstanding shares times the current market price of a stock), and monthly returns are downloaded from Taiwan Economic Journal (TEJ) for constructing six types of portfolios (SH, SL, SM, BH, BL, and BM) then calculating the three factors (MRP, SMB, and HML) based on Fama-French method (1992, 1993, 1995), to analyze the variation of stock returns for each type of portfolio. The results indicate that the explanatory power of the three factors can accomplish about 93% to 97% of a portfolio's return in Taiwan stock market.

Furthermore, the two one-sided tests (TOST) is adopted in this study to investigate the hypothesis of equivalence for these risk parameters, different from the traditional point test. The TOST is first used in the parameter estimates in regression model for analyzing the variation of risk parameters in this study.

We know that the variation of risk is difficult to measure especially in stock market. If the variation of a risk parameter can be further estimated, it usually can help investors to hedge or arbitrage in stock market. No matter hedge or arbitrage, the reason behind, is risk. The reason for hedge is to avoid risk; the reason for arbitrage theoretically is no risk. The source is all from risk, so that the interval of a risk parameter is crucial, and the TOST can help construct the interval estimation of the risk parameter. An empirical method from the concept of risk and return is used to determine the equivalence margin

[1] MRP is market risk premium which is in the sense of expectation. However, the Fama-French adopted "excess market return" to run regression and so did I.

(EM) of a risk parameter in the hypothesis which will be tested by the TOST- induced equivalence region (ER) for each type of portfolio. The comparisons of the calculated risk confidence interval (ER) and the empirical determined EM will provide investors more economically significant meaning about the trade-off between risk and return for each portfolio than traditional point test.

Before the above work is done, the basic statistical assumptions on data have to be checked in section 3 for linearity, normality, un-correlation, and homoscedasticity about errors for an ordinary least squares (OLS) in regression model to work.

The remainder of this study is organized as follows. Section 2 discusses the previous research. Section 3 presents research methods including the data analysis, the model, the development of hypotheses and the equivalence testing. Section 4 summarizes and concludes this paper.

2. Previous Research

The capital asset pricing model (CAPM) has been studied by many scholars (Treynor, 1962; Sharpe, 1964; Lintner, 1965; Mossin, 1966), which is based on the idea that there is a linear relationship between the expected return of a security and the market risk premium.

The CAPM is defined by $E(R_i) = R_f + \left[E(R_m) - R_f \right] \times \beta_i$, where the systematic risk β will be estimatedby a market model and the β is the only parameter used to explain the expected return of the security, which is a simple linear regression model given by $R_{it} = \alpha + \beta \times R_{Mt} + \varepsilon_{it}$.

Ross (1976) presented the Arbitrage Pricing Theory, arguing that there should have k factors to explain the returns, although he did not state what these are. Fama and French (1992, 1993, 1995) extended the CAPM to a three-factor model based on an empirical analysis using historical U.S. stock market returnsand they showed that the average return of a portfolio will be affected by additional risk factors, in addition to the market systematic risk, β .

Fama and French (1993) carried out empirical analysis based on the U.S. stock market and found that the returns of a security will be mainly affected by three factors:the market factor (MRP) from the traditional CAPM, company size (SMB), and book-to-market equity (HML). Fama and French (1993) first constructed six portfolios to form the SMB and HML, then used regression analysis based on MRP, SMB, and HML to build a three-factor model, and the results showed that it could be between 83% and 97% of the variation in stock returns of a portfolio in U.S. stock market. The Fama-French three-factor model consists of a set of three parameters $(\beta_{MRP}, \beta_{SMB}, \beta_{HML})$,which correspond to the three risk factors affecting a portfolio's return. The Fama-French Three-factor model will be stated in (Model 1).

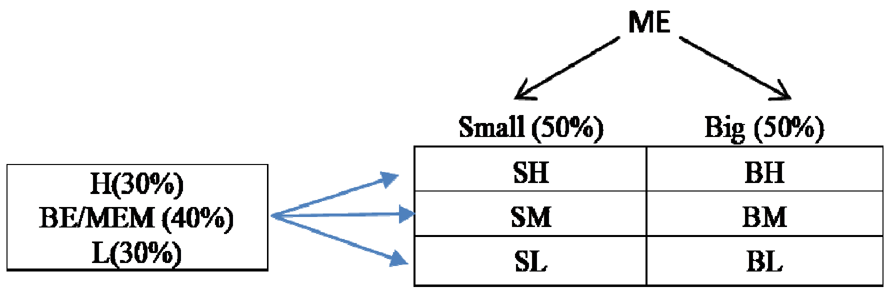
3. Research Method

3.1. Data Description

This study uses book equity (BE), market equity (ME), and the monthly returns of the sample period from July 1982 to December 2012 which was downloaded from the TEJ (Taiwan Economic Journal) to construct six types of portfolios and then calculating SMB, HML, and MRP according to the method proposed by Fama and French (1992, 1993, 1995), and then examines the relations between the rates of return for each of six types of portfolios and three risk factors in Taiwan Stock Market.

The six types of portfolios are monthly value-weighted returns, calculated from July of year *t* to June of year *t*+1 and the portfolios are reformed at the end of each June; they are formed based on size (market equity, ME: small and big) and the ratio of book equity to market equity (BE/ME: high, medium, and low). The sample period was from July 1982 to December 2012, with a total of 366 monthly data, including all Taiwan stocks from listed and publicly traded companies, not including those in the financial and securities industries, totally 199517 observations. The size breakpoint for year *t* is the median Taiwan market equity (ME) at the end of June of year *t*. BE/ME for June of year *t* is the book equity (BE) for the last fiscal year end in year *t*-1 divided by ME for December of year *t*-1. The BE/ME breakpoints are the 30th and 70th percentiles of Taiwan stocks.The Figure 1. is the formation of the six types of portfolios, SMB and HML.

Figure 1: The constructions of six stock portfolios, and the calculation of SMB and HML (Fama-French, 1993, 1995)



The definitions of six types of portfolios are as follows: BH is the portfolio with a "big" size and a "high" ratio of book equity to market equity; BL is the portfolio with a "big" size and a "low" ratio of book equity to market equity; BM is the portfolio with a "big" size and a "medium" ratio of book equity to market equity; SH is the portfolio with a "small" size and a "high" ratio of book equity to market equity; SL is the portfolio with a "small" size and a "low" ratio of book equity to market equity; SM is the portfolio with a "small" size and a "medium" ratio of book equity to market equity.

3.2 The Model

The Fama-French three-factor model is now defined by

$$Y_{it} = \alpha_i + \beta_{i,MRP} \times MRP_t + \beta_{i,SMB} \times SMB_t + \beta_{i,HML} \times HML_t + \varepsilon_{it} \qquad \text{(Model 1)}$$

where

$Y_{it} = R_{it} - R_{ft}$ is the excess return of the i^{th} portfolio at time t,

$MRP_t = R_{mt} - R_{ft}$ is the excess market return at time t,

$SMB_t = R_{St} - R_{Bt}$ is the difference between the average returns on the three small-stock portfolios (SL, SM, and SH) and the average returns on the three big-stock portfolios (BL, BM, and BH) at time t,

$HML_t = R_{Ht} - R_{Lt}$ is the difference between the average returns on the two high-BE/ME portfolios (SH and BH) and the average returns on the two low-BE/ME portfolios (SL and BL) at time t, and

R_{it} is the rate of return of the i^{th} portfolio, i = BH, BL, BM, SH, SL, and SM at time t,

R_{ft} is risk-free rate at time t,

R_{mt} is market return at time t,

β_{ip} is the sensitivity of the p^{th} factor associated with the i^{th} portfolio, p = MRP, SMB, HML ,

α_i is the abnormal return of the i^{th} portfolio, and

ε_{it} is the disturbance term of the model and $E(\varepsilon_{it})=0$ for all $t=1, 2, \ldots, n$, $\text{var}(\varepsilon_{it})=\sigma_{ii}$, $i = 1, 2, \ldots, k$, and $\text{cov}(\varepsilon_{it}, \varepsilon_{ij})=0$ for all , $j, t = 1, 2, \ldots, n$, $j \neq t$.

Therefore, the three factors multiple linear regression models for the six types of portfolios can be written, respectively, as follows:

1. BH

$$Y_{BHt} = \alpha_{BH} + \beta_{BH,MRP} \times MRP_t + \beta_{BH,SMB} \times SMB_t + \beta_{BH,HML} \times HML_t + \varepsilon_{BHt}$$

where $Y_{BHt} = R_{BHt} - R_{ft}$(Model 2)

2. BL

$$Y_{BLt} = \alpha_{BL} + \beta_{BL,MRP} \times MRP_t + \beta_{BL,SMB} \times SMB_t + \beta_{BL,HML} \times HML_t + \varepsilon_{BLt}$$

where $Y_{BLt} = R_{BLt} - R_{ft}$ (Model 3)

3. BM

$$Y_{BMt} = \alpha_{BM} + \beta_{BM,MRP} \times MRP_t + \beta_{BM,SMB} \times SMB_t + \beta_{BM,HML} \times HML_t + \varepsilon_{BMt}$$

where $Y_{BMt} = R_{BMt} - R_{ft}$(Model 4)

4. SH

$$Y_{SHt} = \alpha_{SH} + \beta_{SH,MRP} \times MRP_t + \beta_{SH,SMB} \times SMB_t + \beta_{SH,HML} \times HML_t + \varepsilon_{SHt}$$

where $Y_{SHt} = R_{SHt} - R_{ft}$ (Model 5)

5. SL

$$Y_{SLt} = \alpha_{SL} + \beta_{SL,MRP} \times MRP_t + \beta_{SL,SMB} \times SMB_t + \beta_{SL,HML} \times HML_t + \varepsilon_{SLt}$$

where $Y_{SLt} = R_{SLt} - R_{ft}$(Model 6)

6. SM

$$Y_{SMt} = \alpha_{SM} + \beta_{SM,MRP} \times MRP_t + \beta_{SM,SMB} \times SMB_t + \beta_{SM,HML} \times HML_t + \varepsilon_{SMt}$$

where $Y_{SMt} = R_{SMt} - R_{ft}$(Model 7)

The above regression models will be used to theoretically and empirically analyze the six types of portfolios as follows.

3.3 Data Analysis

Usually economic or financial data may not satisfy the basic statistical assumptions of linearity, normality, uncorrelation, and homoscedasticity with regard to the error terms in an ordinary least squares (OLS) regression model. OLS estimates are thus known as BLUEs- Best, Linear, and Unbiased Estimates. They are "best" because they have the least variance among all other linear estimates (Ayyangar, 2007). OLS estimates are also a linear function of the observations, and are unbiased for parameters. This sectionfirst examines the data to see if they satisfy thebasic statistical assumptions for the OLS to work.

Table 1 presents a summary of the tests of the four basic assumptions of the OLS regression model.

Table 1: Summary of Results of Testing the OLS Assumptions

Assumptions	Results
Linearity	Yes
No Autocorrelation	Yes
Normality	No
Homoscedasticity	Yes

Although the assumption of normality in Table 1 is not supported, based on the asymptotic theorem (Levin, Lin, and Chu, 2002; Im, Pesaran, and Shin, 2003), the limiting distribution of the following statistic will approach standard normality $N(0,1)$,that is

$$Z = \frac{\hat{\beta}_i - \beta_i}{SE(\hat{\beta}_i)} \xrightarrow{d} N(0,1) \quad (3.3.1)$$

Therefore, the OLS can be used to estimate the regression parameters in (Model 1); furthermore, the critical value of 1.96 at a 2.5% significance level can be used in two one-sided tests to carry out the equivalence testing for the analysis of the Fama and French three-factor model.

3.4 Empirical Analysis

By applying OLS, the parameters in (Model 1) have been estimated as in Table 2 where the three factors (MRP, SMB, and HML) can explain the returns of all six types of portfolios (BH, BL, BM, SH, SL, and SM) with an explanatory power of from 93% to 97%. These three factors are thus important indices with regard to the returns of these portfolios.The estimated values of thealpha's and the beta's of the three factors (MRP, SMB, HML) for each portfolio, and the p-values of all three factors with regard to the returns of each portfolio are listed in Table 2.

Table 2: Parameter Estimates for the six types of portfolios July 1982 to December 2012 (366 months, 199517 obs.)

$$R_{it} - R_{ft} = \alpha_i + \beta_{i,\mathrm{MRP}} \times \mathrm{MRP}_t + \beta_{i,\mathrm{SMB}} \times \mathrm{SMB}_t + \beta_{i,\mathrm{HML}} \times \mathrm{HML}_t + \varepsilon_{it}$$

Variables	Dependent variable: the excess returns of the six types of portfolios					
	Multiple Regression Estimation					
	BH	**BL**	**BM**	**SH**	**SL**	**SM**
Alpha	-0.02410 (0.8632)	-0.08257 (0.4172)	0.14717 (0.2756)	0.09531 (0.3495)	-0.01137 (0.9338)	0.12170 (0.3013
MRP	1.05200*** (< .0001)	0.99815*** (< .0001)	0.93984*** (< .0001)	0.98475*** (< .0001)	1.03860*** (< .0001)	0.96664*** (< .0001)
SMB	0.02118 (0.4320)	−0.02107 (0.2827)	−0.06379** (0.0145)	0.98453*** (< .0001)	1.02678*** (< .0001)	0.92500*** (< .0001)
HML	0.80353*** (< .0001)	−0.35695*** (< .0001)	0.13082*** (< .0001)	0.66320*** (< .0001)	−0.17632*** (< .0001)	0.09053*** (< .0001)
Adj. R^2	0.9581	0.9638	0.9354	0.9776	0.9546	0.9619
OBS	366	366	366	366	366	366

Under each column of the parameter estimates the p-values are in parentheses, and ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

For further testing, this study also adopted annualized monthly return for each portfolio with the three-factor regression model, those data are downloaded from TEJ, and the sample period is from year 1982 to year 2012. The results are showed as Table 3.

Table 3: Parameter Estimates (for three factors) for the six types of portfolios Yearly Data 1982 to 2012 (31years)

$$R_{it} - R_{ft} = \alpha_i + \beta_{i,\mathrm{MRP}} \times \mathrm{MRP}_t + \beta_{i,\mathrm{SMB}} \times \mathrm{SMB}_t + \beta_{i,\mathrm{HML}} \times \mathrm{HML}_t + \varepsilon_{it}$$

Variables	Dependent variable: the excess returns of the six types of portfolios					
Multiple Regression Estimation						
	BH	**BL**	**BM**	**SH**	**SL**	**SM**
Alpha	0.03204 (0.9849)	1.01229 (0.4401)	1.46836 (0.5489)	1.09112 (0.3869)	0.11086 (0.9457)	1.31069 (0.6019)
MRP	1.00624*** (< .0001)	1.01001*** (< .0001)	0.93845*** (< .0001)	0.98146*** (< .0001)	0.97768*** (< .0001)	0.99556*** (< .0001)
SMB	0.11134** (0.0135)	−0.01046 (0.7499)	− 0.11851* (0.0618)	0.93246*** (< .0001)	1.05426*** (< .0001)	0.99564*** (< .0001)
HML	0.87404*** (< .0001)	−0.41807*** (< .0001)	0.16215* (0.0670)	0.65580*** (< .0001)	−0.05208 (0.3657)	0.01440 (0.8701)
Adj. R^2	0.9751	0.9677	0.8971	0.9909	0.9820	0.9572
OBS	31	31	31	31	31	31

Under each column of the parameter estimates the p-values are in parentheses, and ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 3 is the parameter estimates for three-factor model under each portfolio, and the sample period is from year 1982 to year 2012. The results indicate that MRP is still the main factor in affecting each portfolio's return, and the R-square are still high and about the same as those in Table 2 on average.

Most of the factor loadings (beta estimates) in Tables 2 and 3statistically significantly affect the return of each portfolio, but they may not be able to provide more economically significant meaning. For this reason a newly estimated equivalence interval oneach parameter for each portfolio is proposed by two one-sided tests, as this can help investors to better understand the relation between the return of a portfolio and the variation of itsrisk.The results of Table 3 will be used to the calculation of ER with the comparison of equivalence margin (EM) inSection 3.7.

3.5 Interval Hypotheses and Two one-sided Tests (TOST)

3.5.1 Interval Hypotheses

Schuirmann (1987) is the scholar who first proposed the "two one-sided test" and he argued two sets of one-sided interval hypotheses about means, which can be stated as

$$\mathrm{H}_0 : \mu_a - \mu_b \le \theta_1 \text{ or } \mu_a - \mu_b \ge \theta_2 \tag{3.5.1}$$

which indicates inequivalence between μ_a and μ_b ,against

$$\mathrm{H}_1 : \theta_1 < \mu_a - \mu_b < \theta_2 ,$$

which indicates equivalence between μ_a and μ_b ,where μ_a is the true mean of treatment *a* and μ_b is the mean of a control; θ_1 and θ_2 are the specified lower and upper boundaries, respectively. Thus, two one-sided test (TOST) is also called interval test or equivalence test. The interval hypotheses (3.5.1) for $\beta 's$ in the Fama-French three-factor model can be similarly described as

$$\mathrm{H}_0 : \beta_i - \beta_b \le L \quad \text{or} \quad \beta_i - \beta_b \ge U \text{ , an inequivalence,} \tag{3.5.2}$$

against

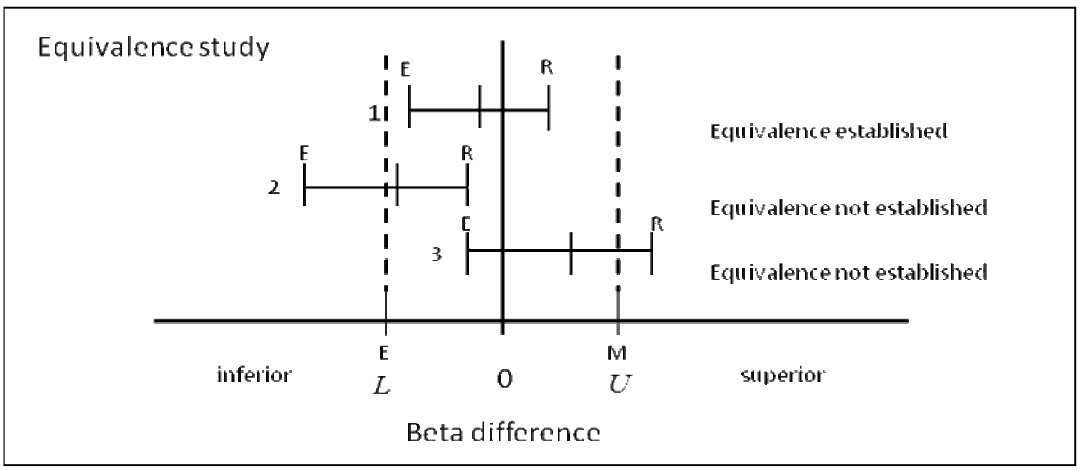
$$\mathrm{H}_1 : L < \beta_i - \beta_b < U \text{, an equivalence ;}$$

where β_i is the risk parameter of each factor and β_b is a specified control value and the benchmark of beta, *L* and *U*are the prespecified lower and upper boundaries, respectively.

The null H_0 in (3.5.2) describes inequivalence between β_i and β_b ,while the alternative states equivalence.This is the reason why the interval hypotheses are decomposed into two sets of interval hypotheses (Stegner, Bostrom, and Greenfield, 1996; Guo, Chen, and Luh, 2011). The goal is to test if the testing objective ($\beta_i - \beta_b$) is located in a predetermined interval. The interval hypothesis means that the equivalence (alternative hypothesis) of risk parameters not only indicates the equality of two betas but also it presents a range for the difference of the testing objectives ($\beta_i - \beta_b$). If the H_1 is hold, then the difference between β_i and β_b is called not much different or saying that β_i and β_b are equivalent.

By modifying its original form, the alternative hypothesis of equivalence is written as a difference of betas, and by definition the difference iswithin the lower limit and the upper limit.

Figure 2: Equivalence vs. Inequivalence



Source: Walker and Nowacki (2011)

3.5.2 Two One-Sided Tests

The test statistics for testing the hypotheses (3.5.2) in the regression model are defined by and

$$Z_U = \frac{\hat{\beta}_i - \beta_b - U}{SE(\hat{\beta}_i)} \qquad \text{(Model 8)}$$

$$Z_L = \frac{\hat{\beta}_i - \beta_b - L}{SE(\hat{\beta}_i)},$$

where $\hat{\beta}_i$ is the parameter estimate of β_i in terms of each type of risk, *U* and *L* are prespecified, and Z_U and Z_L follow approximately standard normal distributions, as previously stated in (3.3.1), by the asymptotic theorem (Levin, Lin, and Chu, 2002; Im, Pesaran, and Shin, 2003). Walker and Nowacki (2011) stated that the most widely used approach to test the hypotheses of inequivalence and equivalence are the use of two one-sided tests (TOST) for treatments.The hypothesis of equivalence H_1 is stated as the difference of the beta from its control falling within the lower limit (*L*) and the upper limit(*U*). Using TOST, the equivalence is established at the α level of significance andhence, a (1–2α) × 100% confidence interval (ER: equivalence region)can be derived by the test. The interval (*L, U*)is called equivalence margin (EM) (Figure 2). The confidence coefficient of the confidence interval, ER, is (1–2α) × 100% , but not the usual (1- α) × 100% beingdue to Bonferroni inequality. A 0.025 significance level for testing the equivalence yields a 95% joint confidence interval.

Using the critical value of 1.96 at a 2.5% significance level, one can reject H_0 if

$$Z_U < -z_\alpha \text{ and } Z_L > z_\alpha \tag{3.5.3}$$

The above rejection region in (3.5.3) can be translated into an equivalence region (ER) derived as follows:

$$Z_U = \frac{\hat{\beta}_i - \beta_b - U}{SE(\hat{\beta}_i)} < -Z_\alpha = -Z_{0.025} = -1.96 \Rightarrow \hat{\beta}_i - \beta_b + 1.96SE(\hat{\beta}_i) < U$$

and

$$Z_L = \frac{\hat{\beta}_i - \beta_b - L}{SE(\hat{\beta}_i)} > Z_\alpha = Z_{0.025} = 1.96 \Rightarrow \hat{\beta}_i - \beta_b - 1.96SE(\hat{\beta}_i) > L.$$

Therefore, we have obtained the equivalence region (ER) given as

$$\text{ER} = (\hat{\beta}_i - \beta_b - 1.96SE(\hat{\beta}_i), \hat{\beta}_i - \beta_b + 1.96SE(\hat{\beta}_i)) \text{ (Model 9)}$$

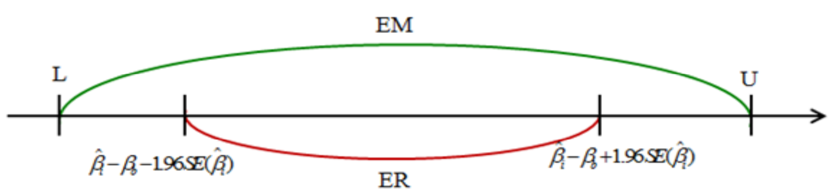
where β_b is the control value for β_i where i=MRP, SMB, HML.

Thus, *if* $\hat{\beta}_i - \beta_b - 1.96SE(\hat{\beta}_i) > L$ and $\hat{\beta}_i - \beta_b + 1.96SE(\hat{\beta}_i) < U$, then

ER for $(\beta_i - \beta_b)$ is within the EM= (L,U) ,

where the equivalence margin, EM= (L,U) , is the largest equivalence interval boundaries. The relation between ER and EM is depicted in Figure 3.

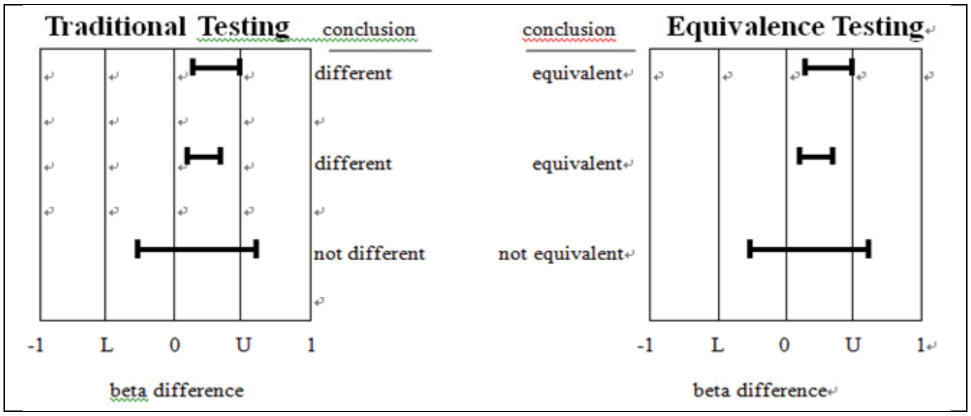
Figure 3: The description of ER and EM



ER $= (\hat{\beta}_i - \beta_b - 1.96SE(\hat{\beta}_i), \hat{\beta}_i - \beta_b + 1.96SE(\hat{\beta}_i))$
EM $= (L, U)$
Thus, *if* $\hat{\beta}_i - \beta_b - 1.96SE(\hat{\beta}_i) > L$ and $\hat{\beta}_i - \beta_b + 1.96SE(\hat{\beta}_i) < U$
then ER is contained within EM. $\Rightarrow$ support H_1

One can reject the null hypothesis of inequivalence if the ER (Model 9) is included in the equivalence margin, EM= (L, U) , or accept the alternative hypothesis of equivalence. The equivalence region and its comparison with the traditional rejection region are shown in Figure 4.

Figure 4: Traditional Testing $\beta_i - \beta_b = 0$ vs. Equivalence Testing $L < \beta_i - \beta_b < U$



Source: Walker and Nowacki (2011)

What is the difference between traditional testing and equivalence testing? The traditional examines if the difference of β_i and β_b equals zero. If $\beta_i - \beta_b$ equals zero (this point), then they are not different; otherwise, they are different as the figure indicated. On the other hand, the equivalence examines if the difference of β_i and β_b locates in an interval of L and U.If $\beta_i - \beta_b$ is within the interval L and U, then they are equivalent; otherwise, they are not equivalent. Therefore, traditional testing uses one point or a given value to perform the test. By contrast, equivalence testing uses an interval to perform the test.

In traditional point test, we only can see the difference of β risk parameter and its control to equal a given value or not (Figure 4). By contrast, equivalence test is able to present the change of beta risk parameter and its control to some extent or provide more information for β_i and β_b. Most important of all,traditional point test can lead to a meaningless economicstate when the sample size is large.Statistical significance can be made by increasing sample size; however, this kind of statistical significance does not represent economic significance and the value on practical applications.

In this study, equivalence is tested for the difference of β_i from its control β_b to see if it lies in an interval within which the β_i can be considered *equivalent* to β_b. The logic behind the two one-sided tests procedure is that if we can reject $\beta_i - \beta_b \geq U$, and also reject $\beta_i - \beta_b \leq L$, according to the decision rules (3.5.3), then it implies $L < \beta_i - \beta_b < U$, or that β_i is not much different from the control β_b. If the equivalence does exist, the null hypothesis will be rejected by the TOST, even if the sample size is large; theconclusions can be drawn in this way because the parameter of interest is not both statistically and economically significant different from its control when ER is in (L, U).Both the one-point hypothesis test and the TOST are generally giving consistent answers, but the TOST provides more information by the equivalence and inequivalence interval, which can tell "economically significant". This concept is thus used to test the three-factor risk parameters associated with MRP, SMB, and HML, and see if they fall into an equivalence interval.

3.6 Hypotheses Testing

It is interesting to see the changes in the beta risks β_{MRP}, β_{SMB}, β_{HML} associated with the three factors for the six types of portfolios (BH, BL, BM, SH, SL, SM) using TOST with prespecified control values. Applying the concept of Figure 2 to set up the equivalence boundaries for each of the three-factor beta risk parameters β_{MRP}, β_{SMB}, and β_{HML} for each portfolio,then the following hypotheses are developed:

1. For factor MRP

$$H_0: \beta_{i,MRP} - \beta_b \le L_1 \quad or \quad \beta_{i,MRP} - \beta_b \ge U_1, \text{ against}$$
$$H_1: L_1 < \beta_{i,MRP} - \beta_b < U_1, \tag{3.6.1}$$

where $\beta_{i,MRP}$ is the market risk for the i^{th} portfolio,i=BH, BL, BM, SH, SL, and SM, β_b is the benchmark of $\beta_{i,MRP}$, and L_1, U_1 are the equivalence margins (EM) of MRP parameter $\beta_{i,MRP}$ minus the control β_b.

2. For factor SMB

$$H_0: \beta_{i,SMB} - \beta_b \le L_2 \quad or \quad \beta_{i,SMB} - \beta_b \ge U_2, \text{ against}$$
$$H_1: L_2 < \beta_{i,SMB} - \beta_b < U_2 \tag{3.6.2}$$

where $\beta_{i,SMB}$ is the size risk for the i^{th} portfolio, β_b is the benchmark of $\beta_{i,SMB}$, and L_2, U_2 are the equivalence margins (EM) of SMB parameter $\beta_{i,SMB}$ minus the control β_b.

3. For factor HML

$$H_0: \beta_{i,HML} - \beta_b \le L_3 \quad or \quad \beta_{i,HML} - \beta_b \ge U_3, \text{ against}$$
$$H_1: L_3 < \beta_{i,HML} - \beta_b < U_3 \tag{3.6.3}$$

where $\beta_{i,HML}$ is the book equity to market equity risk for the i^{th} portfolio, β_b is the benchmark of $\beta_{i,HML}$, and L_3, U_3 are the equivalence margins (EM) of HML parameter $\beta_{i,HML}$ minus the control β_b. For ease of interpretation the equivalence margins (EM) for β_i -β_b (each risk parameter β_i is compared with its control β_b for each portfolio) can be written as $L < \beta_i - \beta_b < U$.

The criteria of the benchmark for each risk parameter used when performing the hypotheses testing are stated as follows. The definition of β_{MRP}, beta, is a measure of the volatility, or systematic risk, of a security or a portfolio in comparison to the market as a whole. Beta is used in the capital asset pricing model (CAPM), a model that calculates the expected return of an asset based on the beta and its associated expected market returns.The β_{MRP} isestimated by OLS using regression analysis, and we can think of the beta as the tendency of a security's returns to respond to swings in the market. The Taiwan Stock Exchange Corporation (TWSE) launched its first Exchange Traded Fund (ETF,tick name is TW50) on June 30, 2003. According to the description of TWSE, ETF is an innovative product of securitized index which measures the trend of the securities market. Investors indirectly invest in the portfolio by holding beneficiary certificates, which represent the index funds. Therefore, investors are able to follow the trend of the index by investing in the ETF, a fund traded on the stock exchange. The ETF consists of the same constituents as the stocks in the index and is divided into smaller trading units.The characteristic of ETF is close to the market; therefore, the beta risk parameter in the ETF is the benchmark for the corresponding risk parameter in a portfolio. If a beta value equals 1 then it indicates that the security's price will move along with the market. If a beta value is less than 1 then it means that the security will be less volatile than the market. If a beta value is greater than 1 then it indicates that the security's price will be more volatile than the market. For example, if a stock's beta is 1.05, it is theoretically 5% more volatile than the market.High-tech-based stocks usually have a beta valueof greater than 1, presenting the chance of a higher rate of return, but also suffering more risk.Utilities stocks in general have a beta value of less than 1. That is the reason why "the risk parameter of ETF" is used as the criteria of the control value for the equivalence hypothesis testing for

β_{MRP} . The same reason applies to β_{SMB} and β_{HML} , so does to the other factors. It should be noted that the upper and lower equivalence boundaries or margins can be non-symmetric (Luzar-Stiffler and Stiffler, 2002).

In the analysis the yearly return of ETF is used, where the yearly return of ETF is downloaded from TEJ where the sample period is from 2003 to 2012 due to the limitation of available sample data period. A multiple regression of ETF return based on three-factor model is run by SAS code to obtain the estimates of risk parameters, which are adopted as the control values of β_b , thus the control values in Table 4 are obtained.

Table 4: The control values of three risk parameters are from ETF in TWSE

Risk parameter	Control value= β_b
β_{MRP}	0.86167
β_{SMB}	-0.04225
β_{HML}	0.05322

3.7 Discussion on the Determination of Equivalence Margins (EM)

Based on Table 3, and theequivalence region(ER) of betas minus their corresponding controls are to be calculated by SAS based on the values of the parameter estimates $\hat{\beta}_i$ by (Model 9) for β_i-β_b as

$$\text{ER}=(\hat{\beta}_i\text{-}\beta_b\ \text{-1.96 SE}(\hat{\beta}_i),\ \hat{\beta}_i\text{-}\beta_b\ \text{+1.96 SE}(\hat{\beta}_i)).$$

It is fascinating to explore how the equivalence margins, EM=$(L,\ U)$ are found. From the concept of risk and return empirically determine the EM. Firstly, taking an example used as explanation. Assuming the market risk premium (MRP) is 8%, and the transaction cost is 0.6% in Taiwan stock market. If someone would like to earn at least an extra return of 0.8% (0.1x0.08=0.008=0.8%) from MRP, then the beta has to rise an additional amount of 0.1, and if one also wants to at least cover the transaction cost of 0.6%, then how much does the beta need to rise? The answer is 0.075. The reason is given below.

$$\frac{0.8\%}{8\%}=0.1 \Rightarrow \text{as } \beta_{MRP} \uparrow 0.1 \text{ then earn } 0.8\% \Rightarrow \text{at least earn } 0.8\%$$

$$\frac{0.6\%}{8\%}=0.075 \Rightarrow \text{as } \beta_{MRP} \uparrow 0.075 \text{ then earn } 0.6\% \Rightarrow \text{at most earn } 0.8\%\text{+}0.6\%\text{=}1.4\%$$

$$\Rightarrow 0.1+0.075=0.175, \text{ the range of } \textit{increment} \text{ in } \beta_{MRP} \text{ is } (0.1,\ 0.175).$$

The earning range from 0.8% to 1.4% can at least cover the transaction cost of 0.6% plus earning an extra return of .8%. Since the range of increment in β_{MRP} is also equal to the range of the beta from its control, $\beta_{MRP}-\beta_b$, thus the equivalence margin of $\beta_{MRP}-\beta_b$ is EM= (0.1, 0.175). If the range of increment of β_{MRP} comparing with the market (the control β_b) is EM= (0.1, 0.175), then the variation of β_{MRP} can at least cover the transaction cost of 0.6%. In other words, the EM for β_{MRP} = (0.1, 0.175) is the largest equivalent risk boundary for the portfolio.Thus, usingthe above empirical concept to determine the equivalence margins, EM=$(L,\ U)$ for the six types of stock portfolios.If the calculated ER based on TOST is within the empirical EM, then the variation of $\beta_{i,MRP}$ moves along with the market (where the control value of β_b stands for the market).So do the other factors.

Secondly, each of the three risk factors is downloaded from TEJ, the sample period is from year 1982 to year 2012, and the mean of each factor is calculated by SAS, the results are shown as in Table 5.

Table 5: The mean of each factor over 31 years

Factors	MEAN (Yearly)
MRP	12.160%
SMB	2.304%
HML	5.827%

The following uses the above empirical method to determine the EM for each beta risk parameter.

1) Mean of MRP=12.16%

Assume β_{MRP} rises 0.1, the extra return is 0.1x12.16%=1.216% and the transaction cost is 0.6%, one wishes to at least cover the transaction cost of 0.6%, what is the equivalence margin (EM) of β_{MRP} from its control? The answer is given below.

$$\frac{1.216\%}{12.16\%} = 0.1 \Rightarrow \text{as } \beta_{MRP} \uparrow 0.1 \text{ then earn } 1.216\% \Rightarrow \text{at least earn } 1.216\%$$

$$\frac{0.6\%}{12.16\%} = 0.049 \Rightarrow \text{as } \beta_{MRP} \uparrow 0.049 \text{ then earn } 0.6\% \Rightarrow \text{at most earn } 1.216\%+0.6\%$$

=1.816%

$\Rightarrow 0.1 + 0.049 = 0.149,$ the *equivalence* margin (EM) of β_{MRP} *from its control* is EM=(0.1, 0.149).

The earning range from 1.216% to 1.816% can at least cover the transaction cost of 0.6% plus an extra return. The control value of β_b for β_{MRP}=0.86167 in Table 4. The EM (0.1, 0.149) is the interval of β_{MRP} comparing with its control β_b, which is the largest equivalent risk interval boundary for each portfolio since the EM has already covered the transaction cost of 0.6% plus an extra return of 1.216%. Thus, any level of risk for each portfolio is equivalent in the EM, and otherwise beyond the EM.

2) Mean of SMB=2.304%

Assume β_{SMB} rises 0.1, the extra return is 0.1x2.304%=0.2304%, and the transaction cost is 0.6%, one wishes to at least cover the transaction cost of 0.6%, what is the equivalence margin of β_{SMB} from its control? The answer is given below.

$$\frac{0.2304\%}{2.304\%} = 0.1 \Rightarrow \text{as } \beta_{SMB} \uparrow 0.1 \text{ then earn } 0.2304\% \Rightarrow \text{at least earn } 0.2304\%$$

$$\frac{0.6\%}{2.304\%} = 0.260 \Rightarrow \text{as } \beta \uparrow 0.260 \text{ then earn } 0.6\% \Rightarrow \text{at most earn } 0.2304\%+0.6\%$$

=0.8304%

$\Rightarrow 0.1 + 0.260 = 0.360,$ the *equivalence* margin (EM) of β_{SMB} *from its control* is EM=(0.1, 0.360).

The earning range from 0.2304% to 0.8304% can at least cover the transaction cost 0.6% plus an extra return. The control value of β_b for β_{SMB}= -0.04225 in Table 4. The EM(0.1, 0.360) is the interval of β_{SMB} comparing with its control, which is the largest equivalent risk interval boundary for each portfolio.Thus, any level of risk for each portfolio is equivalent in the EM, and otherwise beyond the EM.

3) Mean of HML=5.827%

Assume β_{HML} rises 0.1, the extra return is 0.1x5.827%=0.5827%, and transaction cost is 0.6%, one wishes to at least in order to cover the transaction cost of 0.6%, what is the equivalence margin of β_{HML} from its control? The answer is given below.

$$\frac{0.5827\%}{5.827\%} = 0.1 \Rightarrow \text{as } \beta_{HML} \uparrow 0.1 \text{ then earn } 0.5827\% \Rightarrow \text{at least earn } 0.5827\%$$

$$\frac{0.6\%}{5.827\%} = 0.103 \Rightarrow \text{as } \beta_{HML} \uparrow 0.103 \text{ then earn } 0.6\% \Rightarrow \text{at most earn } 0.583\%+0.6\%$$

$$=1.1827\%$$

$\Rightarrow 0.1 + 0.103 = 0.203,$ the *equivalence* margin (EM) of β_{HML} *from its control* is EM=(0.1, 0.203).

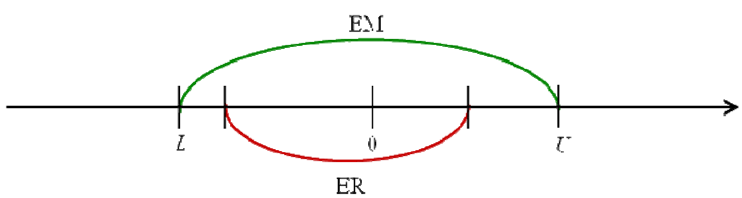
The earning range from 0.5827% to 1.1827% can at least cover the transaction cost of 0.6%. The control value of β_b for β_{HML}= 0.05322 in Table 4. The EM (0.1, 0.203) is the interval of β_{HML} comparing with its control, which is the largest equivalent risk interval boundary for each portfolio.Thus, any level of risk for each portfolio is equivalent in the EM, and otherwise beyond the EM. Consequently, the equivalence margin, EM, has been empirically determined as the above for each factor, and the results are shown as the following Tables. By previous derivation, the ER and EM are as follows.

ER = $(\hat{\beta}_i - \beta_b - 1.96SE(\hat{\beta}_i),\ \hat{\beta}_i - \beta_b + 1.96SE(\hat{\beta}_i))$ is the calculated risk confidence interval

EM = (L, U) is the largest equivalent risk boundary

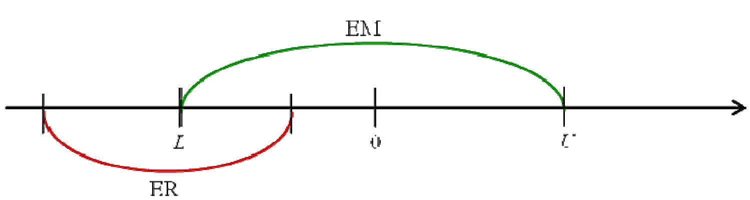
From figure 2 and figure 3, there are three circumstances for ER and EM.

4) The ER is within the EM



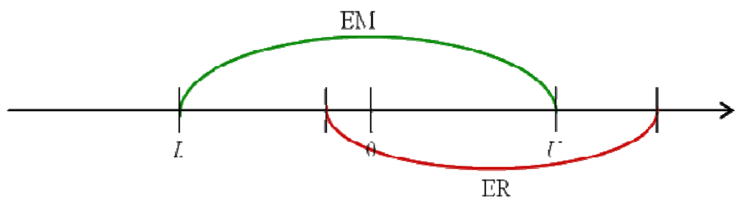
The variation of beta ER is within (L, U), then under this situation, it will not affect investors' investment decision. Because ER being in this interval (L, U) the variation of return is relatively stable, and the variation of β_i from its control β_b will be viewed as being equivalent in the EM= (L, U).

5) The ER is not within the EM (inferior position)



The ER is not within the EM at inferior position- lower risk following lower return since ER is located in the left hand side of EM.

6) The ER is not within the EM (superior position)



The ER sits on the upper side of EM, which cause inequivalence of β_i from β_b .For investors, this will be a good situation for investing when market is up. The variation of beta upper ER boundary is greater than *U*, and the variation of beta lower ER boundary is also greater than *L*, that means investors can make excess return on rising market.

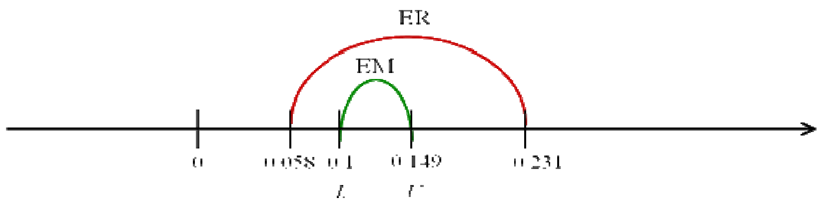
In this study, the framework is from the perspective of investors, assumingan investor would consider one type of stock portfolio according to their investment objective to the six types of portfolios. The ER is pre-calculated beta risk range (i.e. risk confidence interval) by the procedure of the TOST, then the equivalence margin (EM) is the largest equivalent risk range by the procedure of the above empirical analysis. The ER is the risk confidence interval based on the TOST and it might not be within EM, so a risk interval bigger than EM would matter in practice (Julious, 2004). From figure 2 we know that the ER might fall within the EM or might not. Usually if the ER of type A portfolio falls into its EM, then it represents the risk is equivalent to the control for this type of portfolio, so investors can invest in the type A portfolio with less concern; on the other hand, if the ER of type B portfolio in not within its EM, it represents that the risk is larger or smaller. Therefore, the EM is quite like the largest equivalence range of future stock return variation, and the concept of "at least cover the transaction cost" capture the interpretation of the EM in TOST, and the range of EM is determined.

We can think of a portfolio owned by an investor, who would consider his investment decision, at this time the EM is set to be the largest reasonably acceptable risk range, while the ER is the calculated risk confidence interval to be employed to approve EM or disapprove EM.

The equivalence margins, EM=(*L*, *U*) for investorsare like two risk boundaries of an investment, which will be used as guide for considering investing in one of six types of portfolioswhile the portfolio's equivalence region (ER) serve as a decision vehicle in the equivalence testing.In other words, investors viewthis level of stock portfolio's return as not much difference if ER falls into the equivalence margins (*L, U*).

If the calculated equivalence region (ER) from the historical data is contained in the equivalence margins (EM), then investors can invest without concern, and viseversa. In the EM the return is more stable, so the variation of beta is viewed as equivalent; beyond the EM the variation of return is large, so the variation of beta is economically significantly to affect an investor's choice. Consequently,the analysisfor each parameter of each portfolio is obtained and list some of them.

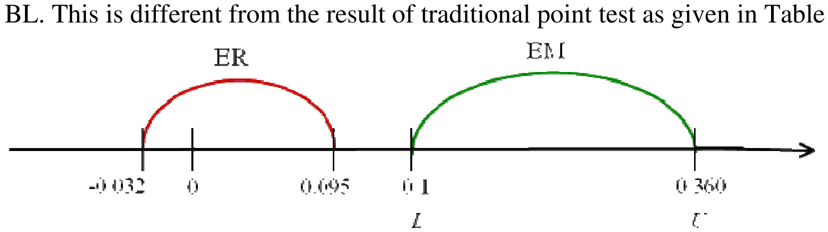
Table 6: Results of testing the equivalence of market parameter β_{MRP} from the control value of ETF(β_b = 0.86167) associated with MRP for portfolio BH

<table>
<tr><td>The Equivalence Region(ER) for β_{MRP} is
$\text{ER} = (\hat{\beta}_i - \beta_b - 1.96\,SE(\hat{\beta}_i), \hat{\beta}_i - \beta_b + 1.96\,SE(\hat{\beta}_i))$
,i=MRP, which varies for each portfolio.</td><td>The equivalence margin (EM) of β_{MRP} is (L_1, U_1)=(0.1, 0.149) for each portfolio according to the calculation in Section 3.7</td></tr>
<tr><td colspan="2">ER of $\beta_{BH,MRP} = (0.058, 0.231)$
The ER=(0.058, 0.231) is not within the EM= (0.1, 0.149), and the ER of portfolio BH has already covered the transaction cost of 0.6%, which implies that portfolio BHwill have higher return than the market in the long run; therefore, $\beta_{BH,MRP}$ statistically and economically significantly affects the return of portfolio BH. This result is consistent with the traditional one point test as shown in Table 3, but providing more information.


The ER= (0.058, 0.231) is not within the EM= (0.1, 0.149), so do not reject H_0.</td></tr>
</table>

The results in Table 6reveal that the EM is a greatest equivalent risk range for each portfolio return, and the ER for each stock portfolio is the individual risk confidence interval; therefore, while the ER is not within the EM that the risk for each stock portfolio either is greater than the market or less than the market. In the case of Table 6, the ER for portfolio BH is not within the EM but will cover the transaction cost 0.6%, thus the return of portfolio BH is higher than the market. Using the equivalence margin as the boundaries of beta risk for each portfolio is a way to decompose the degree of systematic risk for both new investors and existing shareholders. We can clearly see that portfolioBH has a higher return but suffer a higher risk. This is very reasonable for financial investment theory.

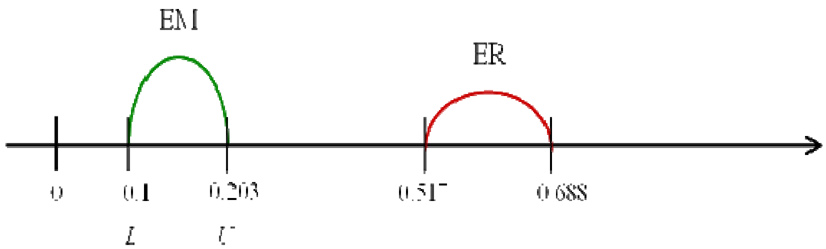
Table 7: Results of testing the equivalence of company size parameter β_{SMB} from the control value of ETF (β_b = -0.04225) associated with SMB for portfolio BL

<table>
<tr><td>The Equivalence Region (ER) for β_{SMB} is
$\text{ER} = (\hat{\beta}_i - \beta_b - 1.96SE(\hat{\beta}_i), \hat{\beta}_i - \beta_b + 1.96SE(\hat{\beta}_i)),\ i = SMB,$
which varies for each portfolio.</td><td>The Equivalence Margin (EM) of β_{SMB} is (L_2, U_2) =(0.1, 0.360) for each portfolio according to the calculation in Section 3.7</td></tr>
<tr><td colspan="2">ER of $\beta_{BL,SMB} = (-0.032, 0.095)$
The ER= (-0.032, 0.095) is not within the EM= (0.1, 0.360) at inferior position, so do not cover the transaction cost of 0.6% in the long run. That implies that the factor-SMB has statistically and economically significantly negative effect to the return of portfolio BL. This is different from the result of traditional point test as given in Table 3.


The ER (-0.032, 0.095) is not within the EM= (0.1, 0.360), so do not reject H_0.</td></tr>
</table>

The ER for portfolio BL, (-0.032, 0.095), is not within the equivalence margin, (0.1, 0.360) at inferior position, and the factor-SMB negatively affects portfolio BLand leads to lower return.SMB is one of the three factors in the Fama and French stock pricing model.As is shown in Table 7 it accounts for the spread in returns between small- and large-sized firms, which is based on the companies' market capitalizations. This is referred to as the “small firm effect”, as smaller firms tend to outperform larger ones. Fama and French’s three-factor model can be used to evaluate a portfolio’s excess returns. SMB can show whether a portfolio manager was relying on the small firm effect (investing in stocks with low market capitalization) to earn abnormal returns. If the manager was buying only small-cap stocks, then the excess returns would be lower than high yielding large stocks, so high yielding large stocks should also be selected.

Table 8: Results of testing the equivalence of BE/ME ratio parameter β_{HML} from the control value of ETF (β_b =0.05322) associated with HML for portfolio SH

<table>
<tr><td>The Equivalence Region (ER) for β_{HML} is
ER=$(\hat{\beta}_i-\beta_b-1.96SE(\hat{\beta}_i), \hat{\beta}_i-\beta_b+1.96SE(\hat{\beta}_i))$,
i=HML, which varies for each portfolio.</td><td>The Equivalence Margin (EM) of β_{HML} is (L_3, U_3) =(0.1, 0.203) for each portfolio according to the calculation in Section 3.7</td></tr>
<tr><td colspan="2">ER of $\beta_{SH,HML} = (0.517, 0.688)$
TheER= (0.517, 0.688) is not within the EM= (0.1, 0.203), but it has already covered the transaction cost of 0.6%. The factor-HML statistically and economically significantly positively affects the return of portfolio SH. The result is consistent with the traditional test as given in Table 3.

The ER=(0.517, 0.688) is not within the EM=(0.1, 0.203), so do not reject H_0.</td></tr>
</table>

The ER (0.517, 0.688) for $\beta_{SH,HML}$ is not within the equivalence margin, (0.1, 0.203), but in a much superior position, so the return of portfolio SH is much positively affected by the HML, and investors may want to choose portfolio SH.

Table 6-8 show the relation of the return of a portfolio between market risk, company size and BE/ME ratio. By using TOST, the equivalence region (ER) for each risk parameter is established, and it is compared with an empirically determined equivalence margins (EM). Then a decision rule can be constructed to decide if a risk parameter is falling into an equivalence margin. By this way TOST brings economic and statistical meaning together at the same time.This is different from the traditional fixed point hypothesis. The following is the summary for the comparison of traditional point test and TOST.

Summary for the Comparison of Traditional Point Test and TOST

Panel A: Factor MRP	$\beta_{BH,MRP}$	$\beta_{BL,MRP}$	$\beta_{BM,MRP}$	$\beta_{SH,MRP}$	$\beta_{SL,MRP}$	$\beta_{SM,MRP}$
Traditional &TOST	O	O	O	O	O	O
Panel B: Factor SMB	$\beta_{BH,SMB}$	$\beta_{BL,SMB}$	$\beta_{BM,SMB}$	$\beta_{SH,SMB}$	$\beta_{SL,SMB}$	$\beta_{SM,SMB}$
Traditional &TOST	O	X	O	O	O	O
Panel C: Factor HML	$\beta_{BH,HML}$	$\beta_{BL,HML}$	$\beta_{BM,HML}$	$\beta_{SH,HML}$	$\beta_{SL,HML}$	$\beta_{SM,HML}$
Traditional &TOST	O	O	O	O	X	X

O indicates that the results of traditional test and TOST are consistent.
X indicates that the results of traditional test and TOST are inconsistent.

4. Conclusion and Implication

Fama-French three-factor model plays a main role in today's financial market around the world, and the model indeed has high explanatory power for the variation of stock return in international financial market. The variation of stock portfolio's return is analyzed in Taiwan stock market by using Fama-French three-factor model, and it indicates that the three factors-market risk, size, and company's book to market ratio are the crucial determination since its explanatory power achieves 93% to 97%. However, it is wondering whether anynew factors can be found for explaining the variation in a stock market in the future, and this merits future research.

Based on the Fama-French three-factor model, the relation between risk and return is further understood by TOST. Risk and return are the leading role in any financial market; the higher the risk the higher the return is as a principle anywhere. Everyone likes highreturn but keeps the risk away, thus financial behavior appears hedge and arbitrage. The source of hedge and arbitrage are risk. For this reason, therisk and return of three-factor, is analyzed by using multiple regression analysis on each type of risk factor for each type of stock portfolio.

Next, applying TOST procedure to perform equivalence test for therisk factors in each portfolio for an appropriate model under study, and this is different from the traditional one fixed-point test. The sense behind this methodology is that the testing result will win the meaning of both statistically and economically significance in essence of the relation between the return of a company and the variation in risk even if the number of observations is getting large. The equivalence region is the confidence interval for each type of risk by adopting TOST, and the determination of equivalence margin is the concept of "at least cover the transaction costs and income taxes" from the empirical analysis of risk and return. In the structure of this study, the characteristic of each stock portfolio is close to a diversified ETF and the value of ETF is close to the market; therefore, the control value (the benchmark of each beta) is determined by the value of theETF. Risk is difficult to measure due to its volatility, for this reason, the determination of equivalence margin is fully discussed in Section 3.7 for each type of portfolio, that is, it is depending on the transaction costs and taxes to interpret risk in the form of equivalence margin.

Traditional hypothesis testing seeks to determine if betas are significantly differentfrom each other (Figure 4), but it can lead to a meaningless statement in economic terms when the sample size is large. In this study, TOSTprocedure is used to test thebeta difference to see if it is lying in an interval within which the betas are considered *equivalent*. Firstly, calculatethe equivalence region (ER) by

SAS, and then apply the empirical analysis of risk and return to construct the equivalence margin (EM) in the frame of TOST.In financial market the mostly used expression is the concept of risk, and the interpretation of the volatility of risk expressed by the equivalence margin in the form of TOST. In this study the equivalence margin is served as the boundaries of an investment, and it consists of the risk upper boundary (*U*) and the risk lower boundary (*L*), and it has three positions of the locations for ER and EM.And the equivalence margin (EM) under each specific risk parameter compared with the pre-calculated equivalence region (ER) is used to determine if the beta risk of each of six types of portfolios is equivalent to a claimed value, thus it can provide a valuable reference to investors. The goal for investors generally is to defeat the market index and make an excess return; therefore, the empirical analysis is employed to interpret the tradeoff of risk and return for explaining the results of equivalence test under each risk factor.

The comparison of the equivalence region (ER) to the equivalence margin (EM) is the key determination of risk level that investors are willing to accept on return. Using the change of stock portfolio's return in relation to the variation of beta, the upper boundary and the lower boundary of the equivalence margin can be obtained. Traditional point test indicates that if each factor affects the return of a portfolio.Equivalence test provides investors with more economically meaningful messages about the trade-off between risk and return for each portfolio than traditional point test.Both the one-point hypothesis test and the TOST are generally giving consistent answers, but the TOST provides more information by the equivalence and inequivalence interval, which can tell "economically significant meaning".

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