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Analyzing Target Redemption Forward Contracts under Lévy Process

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Abstract

The depreciation of the renminbi (RMB) in the last few years had caused many default events on the leveraged structural products called “Target Redemption Forward” (TRF). Analyzing the components of the TRF, we can find these products are composed of buying and selling exchange options. From the empirical analyses of the returns of the exchange rate of USD/CNY, there exist non-normal, leptokurtic and volatility clustering phenomena. Hence, we use the time-changed NIG-Lévy process to construct the dynamics of the exchange rate.Finally, we apply the Monte Carlo simulation technique to price the TRF and analyze the impacts of the clauses in the term sheet of TRF.

Keywords: Target Redemption Forward, volatility clustering, NIG-Lévy, Monte Carlo simulation.
JEL Classification: F31, G12, G13

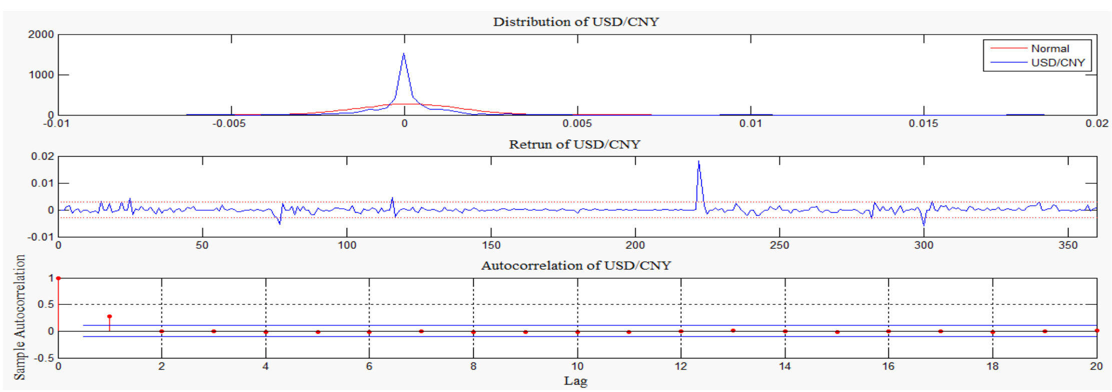
1. Introduction

Target redemption forward contracts (TRF) denominated in renminbi (RMB) have been quite popular due to the expected appreciation of the RMB. However, the RMBdepreciation has led investors to suffer significant losses on TRF, because of theslowing down of GDP growth of Chinaand the impact of the Brexit. At the same time, the banks in Taiwan with higher exposures to TRF face higher default risk from their clients due to the sharp depreciation of RMB during 2015 and 2016.Hence, the exposure control seems extremely important for the banks. Although the exposure can be mitigated through posting collaterals or margin requirement from the clients while their losses over their credit line given by the bank, the amount of collateral or margin should be posted basing on the fair value calculated through model or on quotation from the market. Nevertheless, most of banks still rely on the quotation from the counterparty, especially for complicated structure derivatives such as TRF. While an unexpected event (such as the Brexit) caused the RMB against US dollar to depreciateand resulted in significant losses to clients in TRF positions, the banks without the ability of calculating the price of the TRF and relying on the quotation from the counterparty cannot immediately ask the clients to post collaterals or margin requirement and evaluate the impact of the RMB depreciation.

In traditional studies, the dynamic of the asset prices are assumed to follow a geometric Brownian motion (GBM) proposed by Samuelson (1965) and asset returns are assumed to be normally distributed. From the empirical literature, the asset returns are not truly normally distributed but usually with fat tails and excess kurtosis (Mandelbrot, 1963; Gravin, 2000) caused by jumps or volatility clustering phenomena. Hence, someresearches specified the jump component in the GBM for pricing options ((Merton, 1976, Amin, 1993, and Kou, 2002) and others used the model with stochastic volatilities to enhance the dimensions of the asset processes(Barndorff-Nielsen and Shephard, 2001; Barndorff-Nielsen, Nicolato and Shephard, 2002). Recently, theLévy models are widely discussed in the valuation of derivative securities (Hsu and Chen, 2012; Ornthanalai, 2014; Fajardo, 2015; Jiang et al., 2016) to modify the normal distribution assumption of the GBM. Under the Lévy model, the jump and stochastic volatilities of the asset returns can all be taken into consideration and the phenomena of leptokurtic, fat tail and skew of asset returnscan also be depicted simultaneously. Furthermore, the asset returns in Lévy model with some special specification can be assumed to follow many types of non-normal distributions, such as NIG distribution and VG distribution. In addition, some scholars found that modeling the asset return with Lévy model is better than using the GBM (Schoutens, 2003).

From Figure 1, we can find the returns of USD/CNY exchange rate is not normally distributed but with the phenomena of leptokurtic, jump and volatility clustering. Hence, we use time-changed Lévy process to model the dynamic of exchange rate. The advantage of time-changed Lévy process lies in depicting the jump and volatility clusterting conditions of asset returnsat the same time (Carr and Wu, 2004).

Figure 1: USD/CNY Exchange Rate Return[1]



The remainder of this study is organized as follows. The next section introduces the structure of TRF and the market condition of USD/ CNY exchange rate. Section 3 presents the dynamics of the model. Section 4 provides the numerical experiments and the comparison of TRF contracts. The final section shows the conclusion of this study.

[1] Data period of USD/CNY is 2015/1/1~2015/12/31.

2. The Structure of TRF

The depreciation of the RMB caused many default events on the leveraged products called TRF. Analyzing the components of the TRF, we can find these products are composed of buying and selling exchange options. There are several kinds of TRF contracts in the market and mainly are composed of conditions of contracts being terminated and knocking in (European Knock In, EKI). The terminated conditions can also be divided into three types: (1) terminating while the accumulating points exceeds the knock out point described in the contract; (2) terminating while the accumulating number exceeds the knock out number described in the contract; (3) terminating while the exchange rate touches the knock out price (Discrete Knock Out, DKO). In the issuers of TRF aspects, these terminated clauses being in favor of them protecting the issuers from losing much more money. On the contrary, the setting of the knock in price is in favor of the investors. The investors of TRF suffer the loss only when the fixing rate of the exchange rate exceeds the knock in price. Hence, the combination of terminated conditions and knock in conditions can generate several kinds of TRF contracts such as TRF, TRF with EKI, TRF with DKO and TRF with EKI and DKO. Several kinds of TRF contracts are summarized in Table 1 to Table 4.

Table 1: Type 1 contract of TRF

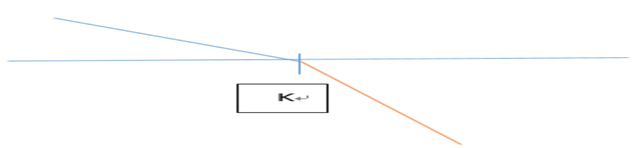
Type 1 TRF There are n settles in these contacts and notional is N. While the fixing exchange rate $Q(t_i)$ is below the strike price K,the investor makes a profit and get $(K - Q(t_i)) \times N$and start to count the accumulatingtimes or point that the investor earns. The contract will be terminated while the accumulating times and point touches the point and times(Target) set in the contracts. While the fixing exchange rate $Q(t_i)$ is over the strike price K, the investor loses $(K - Q(t_i)) \times N \times \text{Leverage}$.
$\text{Payoff} = \begin{cases} N \times [K - Q(t_i)] \text{ if } Q(t_i) \le K \text{ and } \sum_{i=1}^{n} \max(K - Q(t_i), 0) < \text{Target} \\ N \times [K - Q(t_i)] \times \text{Leverage} \text{ if } Q(t_i) > K \text{ and } \sum_{i=1}^{n} \max(K - Q(t_i), 0) < \text{Target} \end{cases}$


Table 2: Type 2 contract of TRF

Type 2TRF with EKI

There are n settles in these contacts and notional is N. While the fixing exchange rate $Q(t_i)$ is below the strike price K, the investor makes a profit and get $(K-Q(t_i))\times N$ and start to count the accumulating times or point that the investor earns. The contract will be terminated while the accumulating times and point touches the point and times (Target) set in the contracts. While the fixing exchange rate $Q(t_i)$ is over the knock in price (EKI), the investor loses $(K-Q(t_i))\times N\times \text{Leverage}$.

$$\text{Payoff} = \begin{cases} N\times[K-Q(t_i)] \ \text{ if } Q(t_i)\le K \text{ and } \sum_{i=1}^{n}\max(K-Q(t_i),0) < \text{Target} \\ N\times[K-Q(t_i)]\times\text{Leverage} \ \text{ if } Q(t_i) > \text{EKI} \text{ and } \sum_{i=1}^{n}\max(K-Q(t_i),0) < \text{Target} \end{cases}$$

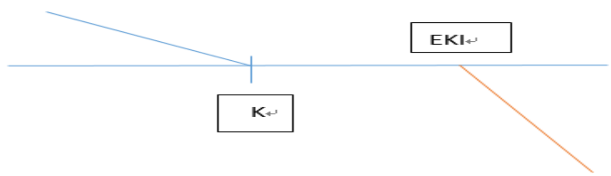


Table 3: Type 3 contract of TRF

Type 3 TRF with DKO

There are n settles in these contacts and notional is N. While the fixing exchange rate $Q(t_i)$ is below the strike price K, the investor makes a profit and get $(K-Q(t_i))\times N$ and start to count the accumulating times or point that the investor earns. The contract will be terminated while the accumulating times and point touches the point and times (Target) set in the contracts. While the fixing exchange rate $Q(t_i)$ is over the strike price K, the investor loses $(K-Q(t_i))\times N\times \text{Leverage}$. In addition, while the fixing exchange rate $Q(t_i)$ is below the DKO price, then the contract is terminated.

$$\text{Payoff} = \begin{cases} K-Q(t_i) \ \text{ if } \text{DKO} < Q(t_i)\le K \text{ and } \sum_{i=1}^{n}\max(K-Q(t_i),0) < \text{Target} \\ (K-Q(t_i))\times\text{Leverage} \ \text{ if } Q(t_i) > K \text{ and } \sum_{i=1}^{n}\max(K-Q(t_i),0) < \text{Target} \end{cases}$$

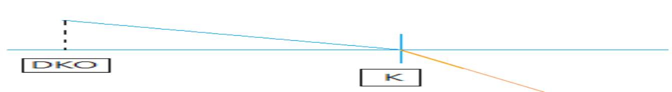
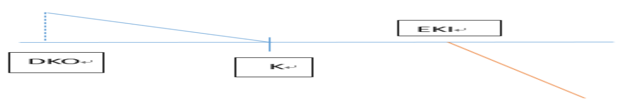


Table 4: Type 4 contract of TRF

Type 4 TRF with EKI and DKO
There are n settles in these contacts and notional is N. While the fixing exchange rate $Q(t_i)$ is below the strike price K, the investor makes a profit and get $(K - Q(t_i)) \times N$ and start to count the accumulating times r or point that the investor earns. The contract will be terminated while the accumulating times and point touches the point and times (Target) set in the contracts. While the fixing exchange rate $Q(t_i)$ is over the knock in price (EKI), the investor loses $(K - Q(t_i)) \times N \times \text{Leverage}$. In addition, while the fixing exchange rate $Q(t_i)$ is below the DKO price, then the contract is terminated.
$報酬 = \begin{cases} K - Q(t_i) \quad \text{if } DKO < Q(t_i) \leq K \text{ and } \sum_{i=1}^{n} \max(K - Q(t_i), 0) < \text{Target} \\ (K - Q(t_i)) \times \text{Leverage} \quad \text{if } Q(t_i) > EKI \text{ and } \sum_{i=1}^{n} \max(K - Q(t_i), 0) < \text{Target} \end{cases}$


3. The Model and Parameter Estimations

3.1 The Specification of the Dynamics of Exchange Rate

In the traditional studies, the asset price is always assumed to follow a GBM model proposed by Samuelson (1965); that is:

$$\frac{dS}{S} = \mu \, dt + \sigma \, dW \tag{1}$$

where

μ: the instantaneously expected asset return,

σ: the instantaneously standard deviation of asset return, and

W: the standard Brownian motion of underlying asset.

We can model the asset price S_t under risk-neutral measure as follows:

$$S_t = S_0 \exp(rt + \sigma W_t - \varpi_{GBM} \, t), \tag{2}$$

where $\varpi_{GBM} = \frac{1}{2}\sigma^2$, denoted as the compensator term of GBM model which ensures that the expectation of $S_t e^{-rt}$ is a martingale under the risk-neutral measure.

Under GBM model, the asset return is normally distributed. However, as mentionedpreviously the distribution of asset return has fat tails and excess kurtosis. In the application of Lévy process, the kurtosis and skewness of the asset return could be depicted in the character of an infinite divisible distribution. Hence, we use the Lévy model to generate the exchange rate. Let X_t be a Lévy process and its cumulantbe $\psi(u)$.[2] According to Lévy-Khintchine formula, we have

$$\psi(u) = ia\,u - \frac{1}{2}\sigma^2 u^2 + \int_{-\infty}^{+\infty}(\exp(iux) - 1 - iux1_{\{|x|<1\}})\Pi(dx)\,, \tag{3}$$

where, $a \in R$, which denotes a linear deterministic part of X_t; $\sigma^2 \geq 0$, which denotes a Brownian part of X_t; and $\Pi(dx)$ is the Lévy measure, which denotes a pure jump part of X_t.

A triplet of Lévy characteristics $[a, \sigma, \Pi(dx)]$ is used to denote the infinitely divisible distribution of Lévy process. Specifically, given the specifications in triplet $[a, \sigma, \Pi(dx)]$, one can determine the distribution type of X_t. Then, the lévy measure of the NIG is in the form of $\Pi_{NIG}(x;\alpha,\beta,\delta) = \pi^{-1}\delta\alpha|x|^{-1}K_1(\alpha|x|)e^{\beta x}$ (Barndorff-Nielsen,1998).

The TRF can be determined based on the simulation of exchange rate Q_t. In Lévy model, the exchange rate under risk-neutral measure is described as follows:

$$Q_t = Q_0 \exp(r_d t - r_f t + X_t - \varpi\, t)\,, \tag{4}$$

where ϖ is the compensator term of Lévy model which ensures the $Q_t e^{-(r_d - r_f)t}$ is a martingale under the risk-neutral measure.

In order to generate the exchange rate and to determine the value of ϖ, we assume the Lévy process X_t follows the specific distribution forms; that is the NIG distribution. If X_t follows NIG distribution (denoted as X_t^{NIG}), then we have

$$X_t^{NIG} = \mu + \beta\, z(t) + \sqrt{z(t)}W\,, \tag{5}$$

where μ is a location parameter; β is the skewness parameter; $z(t)$ follows Inverse Gaussian distribution, denoted as $IG(\delta, \sqrt{\alpha^2 - \beta^2})$[3], α, β and δ are the parameters of NIG distribution ; α (α is the shape parameter). In such case, the compensator of NIG model is described as follows:

$$\varpi = \mu + \delta\gamma - \delta\sqrt{\alpha^2 - (1+\beta)^2}\,, \tag{6}$$

where δ is the excess kurtosis parameter , $\gamma = \sqrt{\alpha^2 - \beta^2}$.

Based on previous descriptions, we can simulate the process of exchange rate in NIG distributions. The simulation procedure is described as follows: (1) we randomly select values of W, $z(t)$ and $G(t)$ from the distribution of $N(0,1)$, $IG(\delta, \sqrt{\alpha^2 - \beta^2})$ and $G(t/v, v)$, respectively; (2) we obtain the value of X_t^{NIG} based on equations (5). (3) we can calculate the value of ϖ according to equation (6). (4) from Equation (6), we can get a simulation path of the exchange rate under the assumption of NIG. With the simulation paths of the exchange rate, then we can calculate different types of TRF values.

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[2] $\psi(u) = \log(\phi(u))$, where $\phi(u)$ is the characteristic function of X_i.

[3] With $\gamma = \sqrt{\alpha^2 - \beta^2}$, then the density of the IG is in the form as

$$f_t^{IG}(x) = \frac{\delta_t}{\sqrt{2\pi}} x^{-3/2} \exp\left(-\frac{\gamma^2}{2x}\left(x - \frac{\delta_t}{\gamma}\right)^2\right).$$

The Method of Parameter Estimations

In this subsection, we use the maximum likelihood method to estimate the parameters ofNIG distribution. The probability density function of NIG distribution is described as follows:

$$f_{NIG}(x;\alpha,\beta,\delta,\mu)=\frac{\alpha\delta}{\pi}\exp(\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu))\frac{K_1(\alpha\sqrt{\delta^2+(x-\mu)^2})}{\sqrt{\delta^2+(x-\mu)^2}}\ , \tag{7}$$

where $x\in R,\delta>0,\alpha>0,0\le|\beta|\le\alpha$; K_1 is the modified Bessel function of the third kind, we have

$$K_1(x)=\frac{x}{4}\int_0^\infty \exp(t+\frac{x^2}{4t})t^{-2}dt\ , \tag{8}$$

The maximum log likelihood function of NIG distribution is as follows:

$$\begin{aligned}&\max_{\theta}\ \ln \mathrm{L}(\alpha,\beta,\delta,\mu\mid x_1,x_2,...,x_n)=\\&\max_{\theta}\left\{(n\log\frac{\alpha\delta}{\pi}+\sum_{i=1}^{n}\left[\delta\sqrt{\alpha^2-\beta^2}+\beta(x_i-\mu)\right]+\right.\\&\left.\sum_{i=1}^{n}\left[\log(K_1(\alpha\sqrt{\delta^2+(x_i-\mu)}))-\log(\sqrt{\delta^2+(x_i-\mu)^2}\right]\right\}\end{aligned} \tag{9}$$

2. Valuation and Comparison of the TRF Contracts

There does not exist closed form solution for evaluating the TRF, because of early termination clauses depending on the accumulated points or times in the previous settlement date. Therefore, we use the Monte Carlo simulation method togenerate 300,000 paths for pricing different types of TRF contracts. The data period for parameters estimation lies between 2015/1/1 and 2015/12/31 and is collected from the Taiwan Economic Journal (TEJ). From the statistical summary presented in Table 4, we can find the returns of USD/CNY exchange rate exist skew and leptokurtic phenomena. The one year risk free rate for the RMB and USD are 2.34% and 0.245%, respectively.

The contracts clauses of the different types of TRF are presented in Table 6 and the fixing schedule is shown in Table 7.Table 5 shows the estimation results of parameters of the NIG model. The exchange rate of USD/CNY in 2016/1/1 was 6.55. From the Table 8, we can find the value of the TRF with EKI clause is higher than the TRF without EKI clause due to the EKI clause is set for protecting the investor from losing much more money. On the contrary, the DKO clause is set in favor of the issuer. Hence, the value of TRF with DKO clause is much less valuable than the rest of the types of TRF contracts. In addition, the leverage also has great influence on the value of TRF contract. The larger the leverage is, the much more money the investor will lose while the exchange rate exceeds the EKI.

From the clauses of the TRF contracts, we also can find there is no protection effect for the investor while the EKI condition is set very close to spot price of the exchange rate. On the contrary, if the DKO condition was set far from the spot price, there still is no protection effect for the issuer. Furthermore, from Figure 2 to Figure 4, we can find the cash flow in each fixing period is in the pattern of decaying. This means the investor will earn money in the initial stage of the contract duration but will lose much more money in the remaining stage till the maturity of the contract due to the leverage effect. This type of cash flow is very similar to thecash flow of another type of financial instrument named as accumulator. Hence, while investing in such complicated financial products, the investor should know the characteristics of the product and the trend of exchange rate. If the exchange rate is on the opposite side as the investor predict, then the investor will lose a great amount of money.

Table 4: Summary of statistics of return of USD/CNY

Currency	USD/CNY
Number of observations	360
Mean	1.27E-04
Standard deviation	1.47E-03
Skewness	5.7927
Kurtosis	71.3288

Table 5: Parameters estimation results

Parameter	Value of estimation(Standard deviation)
α	89.3584(49.4802)
β	48.7639(38.7583)
μ	2.71E-06(1.22E-05)
δ	1.91E-04(1.59E-05)

Table 6: Contracts clauses of different type of TRF

Type of TRF	TRF	TRF with EKI	TRF with DKO	TRF with EKI and DKO
Notional	2 Million	2 Million	2 Million	2 Million
Strike	6.55	6.55	6.55	6.55
Target Point	0.5	0.5	0.5	0.5
EKI		6.7		6.7
DKO			6.3	6.3
Leverage	2	2	2	2

Table 7: Fixing schedule of TRF contracts

	Fixing date	Delivery Date
1	2016/1/31	2016/2/2
2	2016/2/28	2016/3/1
3	2016/3/31	2016/4/2
4	2016/4/30	2016/5/2
5	2016/5/31	2016/6/2
6	2016/6/30	2016/7/2
7	2016/7/31	2016/8/2
8	2016/8/31	2016/9/2
9	2016/9/30	2016/10/2
10	2016/10/31	2016/11/2
11	2016/11/30	2016/12/2
12	2016/12/31	2017/1/2

Table 8: Pricing results of different types of TRF

TRF	TRF With EKI(6.7)	TRF With DKO(6.3)	TRF With EKI(6.7) and DKO(6.3)
-420,519.493	-304,633.20	- 442,440.00	-321,414.70

Figure 2: Value comparison of TRF and TRF with EKI contracts

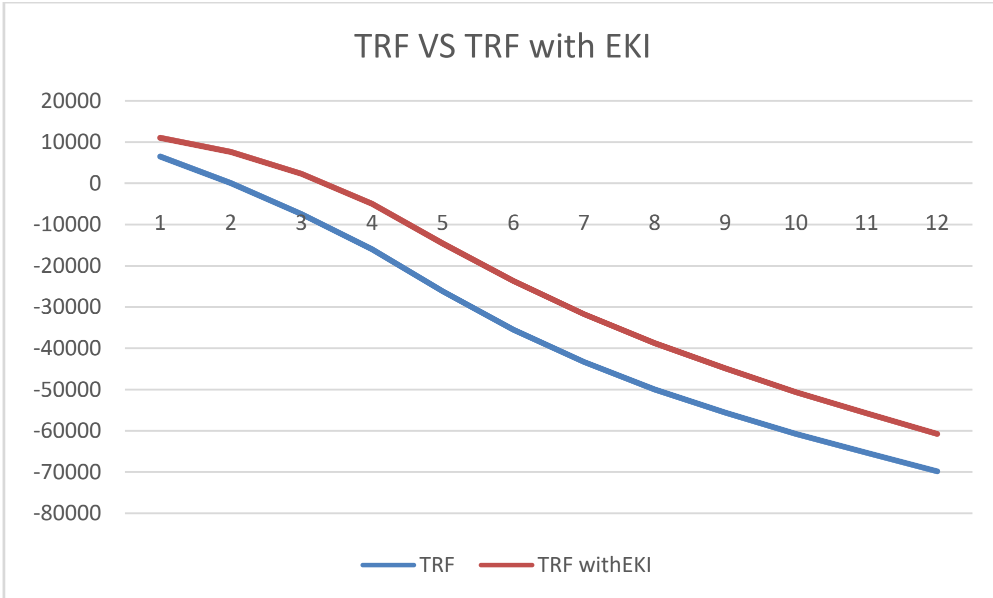


Figure 3: Value comparison of TRF with EKI and TRF with DKO contracts

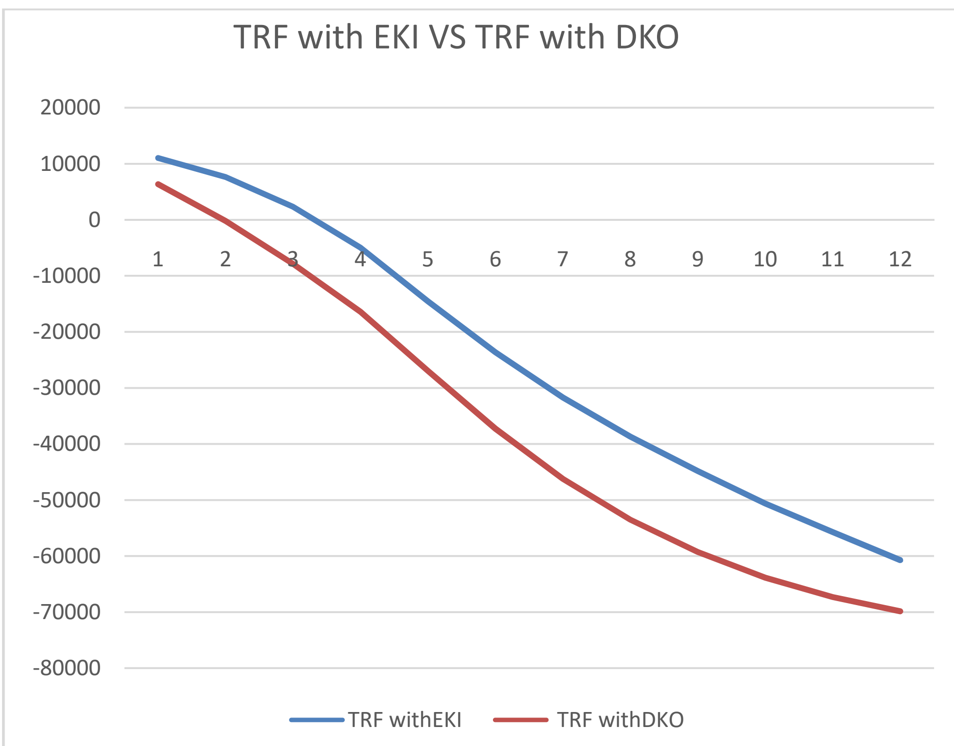
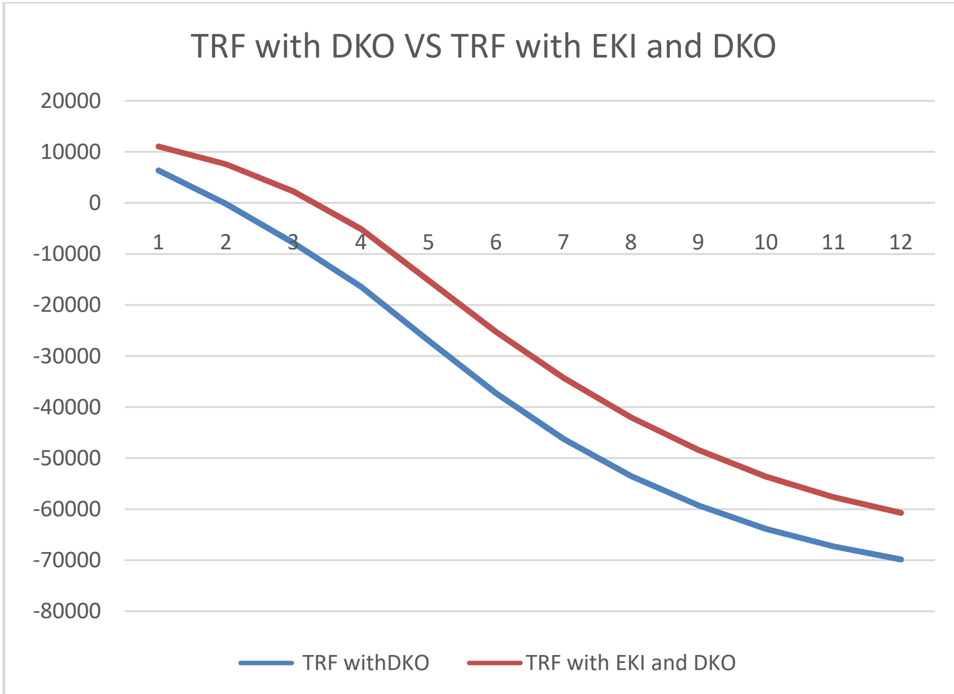


Figure 4: Value comparison of TRF with DKO and TRF with EKI and DKO contracts



Conclusions

The TRF contracts are mainly composed of two mechanisms: (1) protection clause; (2) knock out clause, and combined with different settlement periods and leverage. Although the clauses of TRF contracts are set with protection price (EKI), the investor will lose money very soon while the EKI condition set very close to the spot price. In addition, the leverage in the contracts is almost set as 2, in such case, if the exchange rate exceeds the EKI, the investor will settle the deal with 2 times notional, and this will make the investor accumulate lots of loss in short term rapidly.

Therefore, investing the TRF, the investor should know higher EKI and lower leverage clauses are much more good sake for him while he predicts the currency against USD dollar will appreciate. Furthermore, to forcast the trend of the exchange rate, the investor should also consider the model which can more completely capture the characteristics of exchange rate return. Hence, we use the NIG time-changed Lévy to model the dynamics of the exchange rate to depict the non-normal, jump and volatility clustering phenomena in this study.

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