

Value-at-Risk Under Systematic Risks: A Simulation Approach

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Abstract

Daily Value-at-Risk (VaR) is frequently reported by banks and financial institutions as a result of regulatory requirements, competitive pressures, and risk management standards. However, recent events of economic crises, such as the US subprime mortgage crisis, the European Debt Crisis, and the downfall of the China Stock Market, have provided strong evidence that bad days come in streaks and also that the reported daily VaR may be misleading in order to prevent losses. This paper compares VaR measures of an S&P 500 index portfolio under systematic risks to ones under normal market conditions using a historical simulation method. Analysis indicates that the reported VaR under normal conditions seriously masks and understates the true losses of the portfolio when systematic risks are present. For highly leveraged positions, this means only one VaR hit during a single day or a single week can wipe the firms out of business.

Keywords: Value-at-Risks, Simulation methods, Systematic Risks, Market Risks

JEL Classification: D81, G32

1. Introduction

Definition

Value at risk (VaR) is a common statistical technique that is used to measure the dollar amount of the potential downside in value of a risky asset or a portfolio from adverse market moves given a specific time frame and a specific confident interval (Jorion, 2007). Many financial institutions regard VaR as a standard benchmark for risk management and use it to determine acceptable leverages as well as appropriate consulting strategies (Longin, 2000). Analysts and investors can also use VaR disclosures to compare risk profiles of various trading portfolios (Jorion, 2002).

Assumptions

The simplest estimation of VaR in daily basis makes use of certain assumptions. Under the efficient market hypothesis (EMH) (Fama, 1970), the market prices are expected to follow the random walk

hypothesis, and the returns of each day are independent from each other (Laurence, 1986; Aumeboonsuke and Dryver, 2014). The market returns are also assumed to be normally distributed (Jorion, 2007). However, the studies by Andrew W. Lo and A. Craig Mackinlay (1998) and Jonathan Lewellen (2002) suggest that there are autocorrelations and momenta in stock returns, which would disprove the random walk model with respect to stock price as well as the weak form efficiency theory of the market. The study by Jon Danielsson (2002) points out the dangers of assuming a fixed distribution, such as a normal distribution, to stock returns because the statistical properties of the market are not the same in times of crisis as they are in times of stability. Another assumption is that the portfolio can be rebalanced at any time. In practice, risk managers and investors may not consider changing their asset allocation day by day in response to risk measures (Dryver and Nathaphan, 2012). Furthermore, rebalancing may not be feasible during systematic risks such as market crashes or “the falling sky” (Hellwig, 2009).

Calculation Approaches

Many approaches have been attempted to overcome the weaknesses in the assumptions of simple VaR estimation. C. Brooks et al. (2005), Alexander J. McNeil and Rudiger Frey (2000), Stelios D. Bekiros, Dimitris A. Georgoutsos (2005), Chin Wen Cheong (2008), and I. Khindanova et al. (2001) rely on extreme value theory, various GARCH models and different probability distribution functions such as Pareto, and Cauchy distributions to capture the heavy tail properties in the empirical distribution of returns. Other authors use a mixture of parametric and nonparametric distribution, Markov mixture, or saddle point techniques with Monte Carlo simulation in order to estimate the tail losses (Wong, 2009; Danielsson and De Vries 2000; Haas 2009). Although the final goal is an estimation of possible loss for risky investments within a reasonable bound, different approaches may yield different results for the same portfolio depending on the parameters, assumptions, and methodology (Beder 1995). Another thing to consider is that no matter how the estimation methods are interpreted, VaR does not yield insights into the possible loss beyond the possible threshold values (Yamai and Yoshida 2005), so the actual amount of losses, which is expected to be tolerable based on normal conditions, becomes significant under adverse market movements. This paper focuses on investigating and comparing VaR estimates under normal market conditions and under systematic risks while taking into account the possibility of the large market downfall of October 2008.

2. Technical Notation of Value-at-Risk

The reported daily VaR with a significant level of α means that there is only a probability α that a maximum loss of such reported magnitude or more would occur within a 24-hour time frame (Chan and Wong, 2006). In this paper, simple returns are employed for estimating VaR

$$R_t = \frac{P_t - P_{t-1}}{P_t}$$

where R_t is the rate of return, P_t is the price at date t , and P_{t-1} is the price at the previous day.

Given an initial investment W_0 and the α percentile from the return distribution R_α , the formula for VaR can be expressed as the following:

$$VaR(\alpha) = W_0 R_\alpha$$

VaR is understood as the maximum loss given a significant level α , meaning this paper takes the conventional approach to reporting VaR without the negative sign (Jorion, 2007).

There are many ways to estimate R_α depending on which distribution is being employed to model the returns. The authors take the conventional approach by assuming that the returns are normally distributed with a mean μ and a standard deviation σ in normal market conditions. In a market with systematic risks, the authors combine various normal distributions and streaks with

different estimated parameters to model the return distribution. The population mean and standard deviation are estimated by a sample mean

$$\bar{R} = \frac{1}{n} \sum R_i$$

and a sample standard deviation

$$S_R = \sqrt{\frac{1}{n-1} \sum (R_i - \bar{R})^2}$$

Using these notations, the estimated VaR can be expressed as the following (Chan and Wong, 2006):

$$VaR(\alpha) = -W_0 (\bar{R} + Z_\alpha S_R)$$

where Z_α is the α percentile from the standardized normal distribution $N(0,1)$.

3. Methodology

The authors simulated data in three different ways. The first way was by using a single normal distribution with one mean and one standard deviation. The mean and standard deviation were estimated using the S&P 500 from the beginning of 2005 to the end of the year 2015.

The second model again, the mean and standard deviation were estimated using the S&P 500 from the beginning of 2005 to the end of the year 2015. This scenario, however, used all dates in that time with exception to October 2008 for one of the distributions. The second normal distribution used the data from only October 2008 for the mean and standard deviation. The probability of a typical day's distribution was 97.7% and 2.3% approximately for bad days.

The third model used was simulated using a mixed normal model assuming streaks. In the second model, we assumed the probability of coming to a day that starts at the beginning of an "October 2008", a systemic risk type event, was 1 in 1000, and approximately 1 in 4 years. This could be considered high by some; however, the objective was to be on the cautious side, as risk taking may increase the profitability of banks in good times. Also, as seen in the past when things went wrong, the public came to their rescue, thus a more conservative approach should be taken as a major part of the risk is taken on by the public but not of the profit. Finally, as there were 23 trading days in October 2008, the authors simulated 23 returns for each time the simulation was set off during the systemic risk period. Thus the expected percentage of bad days were the same as in the second model, except now they came in streaks of length 23. Should a streak occur at the end of the specified total number of days to simulate, it will be cut short by 23 days. Thus in essence there was a slightly smaller percentage of bad days in this model than in the second model.

The authors assumed 250 trading days per year. Ten years were simulated 100 times. The authors compared daily VaR and weekly VaR. Daily VaR was calculated in the usual manner. As for weekly VaR, it was calculated using blocks of 5 returns. Thus under the first and second model, VaR could be calculated as the typical VaR calculation under a theoretical scenario of independence. With respect to the third model, however, there would basically be streaks of bad times, and in essence observations of returns were not independent. As a result, this created a much larger potential weekly loss.

4. Simulation Results

The simulation results are listed in Table 1 thru Table 6

Table 1: Daily results under the mixed streaked model with 5%, 50%, and 95% given as the various percentiles investigated

	95%	50%	5%
max	0.16106	0.11738	0.04217
.1%	0.11394	0.0858	0.0371
.5%	0.07274	0.04255	0.03066
1%	0.0484	0.03182	0.02748
5%	0.02186	0.0203	0.01909

Table 2: Daily results under the normal random bad days' model with 5%, 50%, and 95% given as the various percentiles investigated

	95%	50%	5%
max	0.16441	0.12289	0.09351
.1%	0.11185	0.08934	0.06883
.5%	0.06106	0.04539	0.03596
1%	0.03703	0.03221	0.02962
5%	0.02126	0.02033	0.01943

Table 3: Daily results under the normal model with 5%, 50%, and 95% given as the various percentiles investigated

	95%	50%	5%
max	0.05171	0.04389	0.03887
.1%	0.04231	0.03861	0.03559
.5%	0.03477	0.03259	0.03074
1%	0.0311	0.02953	0.0281
5%	0.02165	0.0208	0.01999

Table 4: Weekly results under mixed streaked model with 5%, 50%, and 95% given as the various percentiles investigated

	95%	50%	5%
max	0.32183	0.19837	0.07786
.1%	0.26942	0.17501	0.07423
.5%	0.18374	0.0992	0.06339
1%	0.13245	0.07165	0.05728
5%	0.05137	0.04487	0.03968

Table 5: Weekly results under normal random bad days' model with 5%, 50%, and 95% given as the various percentiles investigated

	95%	50%	5%
Max	0.19102	0.13742	0.10063
.1%	0.16485	0.12754	0.09678
.5%	0.12619	0.09793	0.07768
1%	0.10358	0.0807	0.06752
5%	0.05469	0.04863	0.04368

Table 6: Weekly results under normal model with 5%, 50%, and 95% given as the various percentiles investigated

	95%	50%	5%
Max	0.10447	0.08406	0.0709
.1%	0.09592	0.08013	0.06953
.5%	0.08041	0.07039	0.0623
1%	0.07195	0.06409	0.05742
5%	0.05037	0.04549	0.04135

5. Conclusion and Discussion

As suggested by the simulation results, the VaR measures under normal time, as in model 1, seriously understate the possible losses when the systematic risks are considered, as in model 2 and model 3. At the 5% level, there is not much difference in VaR mean measures and the confident intervals among the three models. When examining beyond this level, the difference becomes remarkable. At the 0.1% level, the VaR mean estimates of models with systematic risks (model 2 and 3) are more than doubled compared to that of the model with normal time (model 1). The 5% confident interval of model 3 is the largest among the three models, which implies a high volatility in the actual realized losses of the portfolio of the S&P500 index.

Streaks of bad days are not uncommon in practice (Dryver 2015), which means that when the systematic risk becomes critical, the market is inefficient. Inefficiency causes huge loss accumulation in the portfolio, which makes rebalancing extremely difficult. With large investment banks, this can be a disaster. According to the Financial Crisis Inquiry Commission (2011), major investment banks in the US were operating with extraordinarily thin capital and less than a 3% drop in asset values could have wiped out the firms. Lehman Brothers was a typical example of “too big to fail” while this bank was operating with 31:1 leverage in 2007 yet filed for bankruptcy in September 2008. Clearly, even a big institution can only withstand a 3%-4% loss. However, the simulation process in this paper, which assumes that the market is at least weak-form efficient, shows that the loss in the presence of systematic risks can go further to 8%-9% in a day and 12-17% in a week. Although the probability of 0.1% for such an event to occur may seem trivial in a day, Dryver, A.L. and S. Nathaphan (2012) showed that under weak-form efficiency, the probability of such an event within one year was 22.28%. This figure rose to 71.65% within 5 years and to 91.96% within 10 years. Under systematic risks such as the US subprime crisis, the EU debt crisis, the China stock downfall, and recently, the Brexit, the market may become inefficient (McInish and Puglisi, 1982; Ojah and Karemera, 1999; Dickinson and Muragu, 1994) and investors should expect that with more frequent VaR hits.

Daily and weekly VaR measures exist for risk management. However, when repeating the processes over and over for years, managers should prepare for the worst case scenarios, even if the probabilities that they occur are small. In addition, bad days are not spread out evenly and often come in streaks. Thus, the actual loss under systematic risks is much larger than one under normal times. The authors recommend that managers consider the possibility of systematic failures of the market as well as determine appropriate leverages and strategies for risk management. Other aspects such as the correlation structures of the returns and the effect of diversification will be considered in future studies.

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