International Research Journal of Finance and Economics
ISSN 1450-2887 Issue 162 July, 2017
http://www.internationalresearchjournaloffinanceandeconomics.com

Liquidity-Adjusted Value at Risk and Hellinger Distance Measure: Evidence from Taiwan Single Stock Futures

Chui Chun Tsai
Accounting Department, Providence University
Sec. 7, Taiwan Boulevard, Taichung, Taiwan
E-mail: cctasi@pu.edu.tw
Tel: +884-04-26328001; Fax: +884-04-27081039

Tsun Siou Lee
Department of Finance, National Taiwan University
Sec. 4, Roosevelt Rd., Taipei, Taiwan
E-mail: tsunsiou@ntu.edu.tw
Tel: +884-02-33661077; Fax: +884-02-23660764

Abstract

This paper empirically investigates the liquidity adjusted Value-at-Risk (LaVaR) of Single Stock Futures (SSFs) using the Hellinger distance measure by sensitizing endogenous liquidity risk with trade sizes at 1%, 5%, and 6%. We find that by incorporating exogenous and endogenous liquidity risk adjusted, LaVaR produces more accurate risk estimates. The practical failure rates of all SSFs are largely consistent with their theoretical failure rates. Despite the use of different empirical models, the highest and lowest LaVaR are CEF and CJF.

Keywords: LaVaR, Hellinger distance measure, Taiwan Single Stock Futures
JEL Classification: D46, D81, G32

1. Introduction

Bank for International Settlements (BIS) classifies risks as belonging to one of five categories: market risk, credit risk, operation risk, legal risk, and liquidity risk (see Table 1).In 1988, to calculate a capital adequacy ratio, BASEL Committee on Banking Supervision (BSBS)stressed the importance of credit risk measures in Basel I Accords. Due to the 1987 U.S. stock market crash,amendment Basel Accord emphasized the importance of establishing a standard statistical model and quantitative analysis for market risk. In 1996, BCBS stressed the importance of market risk measures and required banks to declare their maximum threshold loss under fixed confidence levels and a given time horizon, i.e., Value-at- risk (VaR).[1] (see J. P. Morgan 1996 RiskMetrics measurement; U.S. Securities and Exchange Commission VaR information in 1997 financial statements exposure; VaR measure on market risk of 2004 Basel II Accord).[23] BCBS furthermore introduced three new directives in the 2004 Basel II

1Value-at-risk (VaR) arose in 1993. By 1996, the amended Basel Accord required banks to comply with the contents to calculate VaR thresholds, by evaluating a 5% VaR model over a 12-month test period for 250 trading days (Chen et al., 2012). We take a 5% one-tailed probability and a 95% confidence levelto test consistency between practical violation rates and theoretical proportion of failures.

2 Data for the2004 Basel II Accordsis taken from http://www.bis.org.

Accords to cover exposureoperation risk and legal risk. At the time, the relevant regulations were insufficient to adequately compare liquidity risk exposure and other types of risk. However, a continuous series of financial crises (i.e., 1997 and 2007-2008) resulted in ongoing low liquidity conditions which damaged security markets worldwide. In the face of contagious liquidity risk, the BCBS set rules to regulate liquidity exposure in the Basel III Accords. These measures were still insufficient to adequately determine the liquidity risk effect on value at risk (see the liquidity requirement of the 2010 Basel III Accords)[4].

Table 1: Basel I, II, III and Risk Exposure

Risk	Period
Liquidity risk	2010~2017 (BASEL III)
Operation risk	2004 (BASEL II)
Legal risk	2004 (BASEL II)
Market risk	1996 (BASEL I extended)
Credit risk	1988(BASEL I)

Source: Bulletin Board Download Page on the Basel Committee on Banking Supervision. (Jun. 30, 2016)

As Stoll (2000) pointed out that poor liquidity leads to friction costs in imperfect markets, thus creating a gap between theoretical and real market prices (Stoll, 2000; Berkowitz, 2000). However, traditional VaR assumes the asset only has market risk, while not any liquidity and credit risk. VaR could calculate the threshold loss value at given a specific portfolio, time horizon and one-tailed probability by mark-to-market pricing (Jorion, 2006; Chen et al., 2012; Chang et al., 2016). Bangia et al. (1999, 2001) introduced liquidity-adjusted Value-at-risk (LaVaR), and involved the liquidity risk to adjust the VaR only measure the simple mean-variance at market risk, which is a basic traditional LaVaR model based on the imperfect market and frictional cost hypotheses. They noted that disregarding the liquidity-adjusted VaR would underestimate risk by between 25% and 30%. Compare with traditional VaR assuming the well-liquidity, LaVaR could measure the risk due to the ill-liquidity effects.

Bangia et al. (1999, 2001) also classified liquidity risk into exogenous liquidity risk and endogenous liquidity risk, where the bid-ask spread is used as a proxy variable to measure exogenous risk. The incrementally incurring spread gap is directly related to the exogenous liquidity risk and increased cost of liquidity (COL) of a financial asset on traditional VaR. (Aubier & Saout, 2002; Ourir & Snoussi, 2012) Exogenous liquidity risk leads to mispricing between bid and ask prices, and is seldom caused by individual investors, but always increases overall market price volatility and risk. Investors are typically more concerned with uncontrollable risks and stress the exogenous liquidity risk effect, i.e. they look for spread volatility and appraise the COL. Most investors focus exclusively on the exogenous liquidity risk effect. On the other hand, trade size is a proxy variable used to measure endogenous risk. (Bangia et al., 1999, 2001) The incrementally incurred trade size fluctuation is directly related to the exogenous liquidity risk and increased transaction costs of a financial asset. In particular, higher exchange volumes will sharply increase the spread volatility and COL. Thus, increasing endogenous risk would result in increased exogenous risk in advance and deepen market ill-liquidity in a vicious circle (Demsetz, 1968; Black, 1971a, 1971b; Kyle, 1985; Glosten & Harris, 1988; Stoll, 2000; Simonian, 2011).

Furthermore, in Bangia et al. (1999, 2001) research, they only define the endogenous liquidity risk and explain possible effects on VaR measuring, but did not establish an empirical model and largely neglect its role in their empirical research. Lawrence & Robinson (1998), Häberle & Persson (2000), Aubier (2002), Zhan & Hun (2001), Shen et al. (2002) and Si & Fan (2012) used exogenous

3Data for the U.S. Securities and Exchange Commission VaR information in 1997 financial statements exposure is taken from https://www.sec.gov/divisions/corpfin/guidance/derivfaq.htm

4 Data for the 2010Basel III Accords is taken from http://www.bis.org.

liquidity risk to refine the traditional VaR only as Bangia et al. (1999, 2001). Some early studies incorporated endogenous liquidity risk into LaVaR (e.g., Jarrow & Subramanian, 1997, 2012; Berkowitz, 2000; Subramanian & Jarrow, 2001; Cosandey, 2001; Le Saout, 2002). More recently, Al Janabi (2011a, 2011b, 2013), Tsai & Li (2015), and Tsai & Wu (2016) used trade size as an empirical variable to recalculate the liquidity horizon and measure endogenous liquidity risk effects. Simonian (2011) used trade size percentage (d_H) as a variable to adjust the endogenous liquidity risk effect on traditional LaVaR, assuming a numeric analysis at 1% market size and using the Hellinger distance measure concept in his research, which is a new method to adjust the endogenous liquidity risk on traditional LaVaR.

For using Hellinger distance measure to sensitize the endogenous liquidity risk at different trade size percentages (d_H), we refer to Simonian's (2011) and Bangis et al. (1999, 2001) research. By modifying the endogenous liquidity risk on exogenous LaVaR through two sequential empirical models, which are traditional LaVaR model and endogenous liquidity risk-adjusted LaVaR model, we use 12 Taiwan Single Stock Futures (SSFs) listed on Taiwan Futures Market (TAIFEX) as empirical data, and broaden the scope of considering the different trade size percentages, in which, 1% based on Simonian's (2011) research, and others are the average and maximum percentages of actual trade size in markets.Thus, in Sections 1 and 2, we discuss the concepts of traditional LaVaR and the Hellinger distance measure. By using the Hellinger distance measure, we adopt endogenous liquidity risk to modify the traditional LaVaR only considering exogenous liquidity risk. In Section 3, we describe empirical data through the Taiwan Single Stock Futures (SSFs) and build the empirical models. In Sections 4 and 5, we explain the results of empirical models, and compare the consistency of practical failure rates and their corresponding theoretical failure rates based on the consistency of the ex-post loss and ex-ante VaR according to the back-testing results. We finally suggest implications for SSFs investment decision-making.

2. LaVaR and Hellinger Distance Measure and

The Hellinger distance, also called the Bhattacharyya distance, can be used in metric space to measure the degree of disorder between two sets of probabilities in a d-metric state space Ω, e.g., probability P and Q. It monitors the probability measure space of P and Q. Suppose P and Q are two normal distributions, where $P \sim N(\mu_{1,t}, \sigma_{1,t})$ and $Q \sim N(\mu_{2,t}, \sigma_{2,t})$are absolutely continuous with respect to a σ-finite dominating measure λ. P and Q follow the f-divergence. We use measure theory to quantify the similarity distance between the two relationship probabilities, which are like the relationship probabilities between traditional LaVaR and endogenous liquidity risk-adjusted LaVaR in this research.

According to Borel's measures, we consider P and Q to respectively be the relative entropy that relies on calculating the divergence of D_f (P||Q) in a limited finite measure space σ-field, and the measure value of b-a in a specific interval [a, b] for any third probability measure λ. The distance is entropy that will initially be divergent and disordered. The convergence will progress gradually from the high-entropy phase to the low-entropy phase. The Hellinger distance is defined in terms of the Hellinger integral and we apply this concept to calculate the total variation distance. In our research, the results reported by Bogahev (2007) and the Hellinger distance are used to provide a convenient expression of measures which fall in the range [0, 1] such as b and a. These are the same as the market position percentage of trade sizes between 0% and 100%. The Hellinger distance is expressed in advance as a norm, and the length of the square of distance is a vector's inner product or dot product, that is $H^2(P||Q)$. This can then be written as the total variation distance as $H^2(P_1 \otimes P_2, Q_1 \otimes Q_2) \leq \int H^2(P||Q)$ under two probability outcomes which do or do not consider liquidity risk. It can also be written as entropy, that is $D(P||Q) = \int D(P||Q)$, or as the total variation distance $D(P_1 \otimes P_2, Q_1 \otimes Q_2)$ under the two probability outcomes which do or do not consider liquidity risk. We calculate the distance to determine the probability outcomes under the Lebesgue measure L_1, and the total variation measure can be presented as $||P - Q||_1$. In particular, following the Cauchy-Schwarz inequality, the relation between these two probability distributions is $||P - Q||_1 \leq 2H(P||Q) \leq 2\sqrt{||P - Q||_1}$. The square of Hellinger distance between P and Q is as following:

$$H^2(P||Q) = \frac{1}{2}\int \left(\sqrt{\frac{dP}{d\tau}} - \sqrt{\frac{dQ}{d\tau}}\right)^2 d\lambda$$

where $H^2(P||Q)$ isthe square of Hellinger distance. $\frac{dP}{d\tau}$ and $\frac{dQ}{d\tau}$ are two Radon–Nikodym derivative probability measures and the values won't be changed despite τ being replaced by other probabilities.

For compactness, the above distance measure is usually written as $\frac{1}{2}\int(\sqrt{dP} - \sqrt{dQ})^2$. To define the Hellinger distance in terms of elementary probability theory, τ could be measured using the Lebesgue measure, so that $\frac{dP}{d\tau}$ and $\frac{dQ}{d\tau}$ are simply probability density functions. If P and Q are denoted as the respective densities f and g, the squared Hellinger distance can be expressed as a standard calculus integral, where the second form is obtained by expanding the square and based on the fact that the integral of a probability density over its domain must be one. The Hellinger distance $H^2(P||Q)$ satisfies the property and is derivable in the Cauchy-Schwarz inequality and $0 \leq H(P||Q) \leq 1$, that is as following:

$$H^2(P||Q) = \frac{1}{2}\int \left(\sqrt{f(x)} - \sqrt{g(x)}\right)^2 dx = 1 - \int \sqrt{f(x)g(x)}dx$$

where $H^2(P||Q)$ is also asquare of Hellinger distance. f(x) and g(x) are two probability measures of the state space Ω, which could be used to measure the difference between traditional LaVaR and endogenous adjusted LaVaR at given trade size percentages.

By combining the probability measure of the Hellinger distance characteristics considered in Bogachev's (2007) and Simonian's (2011) research, we apply sensitivity analysis for measuring the endogenous liquidity risk effect. We assume there are two difference probability distance measures between traditional LaVaR considered only the exogenous liquidity risk, and another is involved endogenous liquidity risk adjusted LaVaR. The two probability distribution functions f(x) and g(x) are supposed as the two normal distributions in the state space Ω. As Lagarias et al. (1998) research, the probability distributions are normal distributions (N.D.) and assumed to be absolutely continuous with σ-finite dominating measure, which are $N(\mu_{s1}, \sigma_{s1})$ and $N(\mu_{s2}, \sigma_{s2})$ and as probability distributions of traditional LaVaR and endogenous adjusted LaVaR. The $\mu_{s1,t}$ and $\sigma_{s1,t}$ are the parameters of probability distributions under exogenous liquidity risk (i.e., traditional LaVaR), $\mu_{s2,t}$ and $\sigma_{s2,t}$ are the parameters for probability distributions incorporating endogenous liquidity risk, the probability density functions (PDF) of the normal distributions are respectively

$$f(x|\mu_{s1,t}, \sigma_{s1,t}) = \frac{1}{\sigma_{s1,t}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu_{s1,t}}{\sigma_{s1,t}}\right)^2\right) \text{ and } f(x|\mu_{s2,t}, \sigma_{s2,t}) = \frac{1}{\sigma_{s2,t}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu_{s2,t}}{\sigma_{s2,t}}\right)^2\right).$$

We also regard the probability distance measure between the two probability distributions, i.e., $H(P||Q)$, is as equivalent to the percentage of market trade size (d_H), which is between 0% to 100%. By assuming the Hellinger distance (d_H)at different percentage of trade size, in which, 0% is as traditional LaVaR case as Bangis (1999, 2001), 1% is as Simonian's (2011) research, and others are the average and maximum percentages of actual trade size in markets, we revalue $\mu_{s2,t}$and$\sigma^2_{s2,t}$, and then recalculate the new COL and the endogenous liquidity risk effect on traditional LaVaR.

3. Data and Methodology

3.1 Data

On Jan. 25, 2010, TAIFEX introduced SSFs, the trade size in the initial year was 724,375 units. On May 15, 2014, TAIFEX signed a memorandum with Eurex Group to license TWD-denominated daily futures listed on Eurex, and increased the SSFs listed from an initial year 33 single stock futures to eventually cover all Taiwan-listed securities in 2014, which sharply expanded futures trade size to 9,325,030 units, and increased more than 12 times by 2014. By daily price limits amendments on futures products to 10% in June 2015, also increasing SSFs volume to 12,189,434 units. Market trade size ascended to another 30% between 2014 and 2015. Furthermore, observe the trading data for 2014 to the end of June 30, 2016 (a total of 609 trading observations for each Taiwan-listed securities and SSFs), 12 Single Stock Futures (SSFs) listing on Taiwan Futures Market (TAIFEX), they are Taiwan financial holding companies also among the Taiwan top 50 companies in terms of high market capitalization, with a public float exceeding 5% and stable stock trade sizes in Taiwan Stock Exchange (TWSE). The corresponding ticker symbols of SSFs are respectively CEF, CJF, CKF, CLF, CMF, CNF, DEF, DNF, DOF, DPF, LOF and LRF. Table 2 shown the corresponding underlying stock and financial statements.

Table 2: Taiwan Financial Holding Companies SSFs and Financial Statements

Single Stock Futures (Ticker Symbol)	Underlying Stock	financial statements					
		Asset	Income	EPS	Stock Price		
					High	Low	Average
CEF	Fubon Financial Holding Co.	6160792716	23892514	3.94	69.00	34.70	38.74
CJF	Hua Nan Financial Holdings Co.	2498742336	7690067	1	19.6	13.9	16.6
CKF	Cathay Financial Holding Co.	7807698789	14243235	3.08	56.5	33.6	36.69
CLF	Mega Financial Holding Co.	3343448027	9431773	1.3	13.30	6.94	7.85
CMF	Taishin Financial Holding Co.	1519415261	6701820	0.98	16.2	9.91	12.34
CNF	Chinatrust Financial Holding Co.	4755339067	14306497	1.29	24.80	14.50	16.68
DEF	SinoPac Holdings Co.	1607284236	4982307	0.65	15.10	8.61	9.54
DNF	E.Sun Financial Holding Co.	1844417546	7232972	1.22	22.15	15.85	18.64
DOF	Yuanta Financial Holding Co.	2060989515	6689095	0.9	18.40	9.97	10.49
DPF	First Financial Holding Co.	2488307994	8746263	1.12	22.90	14.15	16.76
LOF	Taiwan Cooperative Financial Holding Co.	3308788132	6890160	0.89	16.40	13.85	14.30
LRF	China Development Financial Holding Co.	893191923	1943281	0.3	13.30	6.94	7.85

Source: Bulletin Board Download Page on the TWSE. (The data analysis is between Jan. 1, 2014 and Jun. 30, 2016.)

Since the individual futures trade size from 2014 onwards has increased substantially, in the same year, TAIFEX introduced almost of all TWSC single stock for market trading, and TAIFEX success to list Taiwan futures on Eurex, therefore, we investigate the SSFs through 12 Taiwan financial holding companies from 2014 to mid-2016. To observe the trading period between 2014 and mid-2016, the trade size percentage ($d_H$hereafter) of Taiwan SSFs is increased,the annual values of 12 SSFs are 5%, 6% and 5%, respectively. The percentage of participants also increased with respectively annual values of 6%, 5% and 5%. Both trade size and participants were obviously stable from 2014 to mid-2016. The maximum value and average market trading percentage of actual trading percentage at 6% and 5%. Thus, by combining the probability measure of the Hellinger distance characteristics, we apply sensitivity analysis to involve the endogenous liquidity riskto adjust the traditional LaVaR assumed the trade size is at 1%, 5% and 6%, which are respectively the percentages as Simonian's (2011) research, average and maximum trading size of Taiwan SSFs.

By using the traditional LaVaR model further, which is a simple and practical model that simultaneously measures asset pricing of mean-variance and adjusts for exogenous liquidity risk. LaVaR model applies the return rate of SSFs to measure the traditional VaR assumed under the market risk, and applies bid-ask mean spread as the proxy variable for measuring the exogenous liquidity risk and adjusting the traditional VaR, i.e., cost of liquidity (COL). The empirical datasets are return rate and bid-ask mean spread, respectively. As in previous studies, we assume the probability distributions of returns and spreads follow a normal distribution. They are $N(\mu_{r,t}, \sigma_{r,t})$ and $N(\mu_{s1,t}, \sigma_{s1,t})$. We respectively calculate the return rate ($\mu_{r,t}$) and mean spread ($\mu_{s1,t}$) by Models1 and 2.

$$\mu_{r,t} = \ln\left(\frac{P_t}{P_{t-1}}\right), \qquad \text{(Model 1)}$$

$$\mu_{s1,t} = \frac{P_{b,t} - P_{a,t}}{\frac{P_{b,t} + P_{a,t}}{2}}, \qquad \text{(Model 2)}$$

where Model 1 is the one-day holding horizon return, P_t and P_{t-1} are the daily closing price in periods t and t-1. Model 2 is the one-day holding horizon bid-ask mean spread, $P_{b,t}$ and $P_{a,t}$are bid price and ask price in period t.

Due to the heterogeneous volatility of the return and mean spread, we built Bollerslev's (1986) GARCH models to estimate $\sigma^2_{r,t}$ and $\sigma^2_{s1,t}$. The empirical models are described as Models3and 4:

$$\sigma^2_{r,t} = \alpha + \beta\sigma^2_{r,t-1} + \gamma\varepsilon^2_{r,t-1}, \qquad \text{(Model 3)}$$

$$\sigma^2_{s1,t} = \pi + \theta\sigma^2_{s1,t-1} + \tau\varepsilon^2_{s1,t-1}, \qquad \text{(Model 4)}$$

where Model 3 is the one-day holding horizon volatility of return, $\sigma^2_{r,t}$, $\sigma^2_{r,t-1}$ and $\varepsilon^2_{r,t-1}$ are respectively the daily closing volatility and residual in periods t and t-1. Model 4 is the one-day holding horizon volatility of mean spread, $\sigma^2_{s1,t}$, $\sigma^2_{s1,t-1}$ and $\varepsilon^2_{s,t-1}$ are respectively the daily mean spread volatility and residual in periods t and t-1.

We then involve the $\sigma^2_{r,t}$ and $\sigma^2_{s1,t}$ estimated results into Models 5 and 6 respectively to measure the traditional VaR and COL, and then incorporate traditional VaR and COL together as Model 7 to calculate the exogenous liquidity risk effect and traditional LaVaR. We apply Models8 and 9 to sensitize the different percentage of trade size effect at 1%, 5% and 6% relatively to 0% assumed, and revalue the μ_{s2} and $\sigma^2_{s2,t}$ by Hellinger distance measure. Adopt the new μ_{s2} and $\sigma^2_{s2,t}$, we measure the new COL as Model 10, and recalculate the endogenous liquidity risk effect on traditional LaVaR as Model 11 further. The empirical models are presented as Models1 to 11 in sections 3.2 and 3.3, including LaVaR model and the sensitizing analysis endogenous liquidity risk model using the Hellinger distance measure.

3.2 Traditional LaVaR

The traditional LaVaR model simultaneously measures the asset pricing of mean-variance under the traditional VaR, and further adjusts for exogenous liquidity risk under the COL. In the first model, we build traditional VaR based on Bangia et al. (1999, 2001) and Simonian's (2001) empirical model, which only considers market risk without considering exogenous liquidity or endogenous liquidity, assuming the cost of liquidity and the trade size percentage are both zero. The measure calculation is expressed as Model 5:

$$\text{VaR} = P_t * (1 - \exp(-1.96 * \eta_{r,t} * \sigma_{r,t})), \qquad \text{(Model 5)}$$

$$\eta_{r,t} = 1.0 + \varphi * \ln\left(\frac{\kappa_{r,t}}{3}\right),$$

where Model 5 is the one-day holding horizon of traditional VaR. P_t is the daily closing price in period t, $\sigma_{r,t}$ is Std. Dev. of the return derived by Model 3. For precisely estimating the risk, we use the correction factor parameter "$\eta_{r,t}$" to modify the bias due to the non-normal distribution (non-N.D.), e.g., "leptokurtic" or "fat-tailed" of theprobability density functions of returns. "$\kappa_{r,t}$" and "φ" are respectively the kurtosis coefficient and one-tailed probability 1%. When parameter $\eta_{r,t}$ is equal 1 and $\kappa_{r,t}$ is equal to 3, the PDF of returns are N.D. and no adjustment are needed; When parameter $\eta_{r,t}$ and

$\kappa_{r,t}$ are respectively greater than 1 and 3, the PDF of returns deviate significantly from normality and the adjustment is needed. (See Bangia et al. (1999, 2001))

We then integrate the exogenous liquidity effect and calculate the cost of liquidity. As adjusted by Bangia et al. (1999, 2001) and Simonian (2001), the COL formula is expressed as Model 6:

$$\mathrm{COL} = \frac{P_t}{2} * \left(\mu_{s1,t} + a * \sigma_{s1,t}\right), \quad \text{(Model 6)}$$

where Model 6 isthe one-day holding horizon of cost of liquidity. P_t is the daily closing price in period t, $\mu_{s1,t}$ is the mean spread derived by Model 2, and $\sigma_{s1,t}$ is Std. Dev. of the mean spread derived by Model 4. "a" is the scaling-adjusted parameter for modifying the bias due to non-N.D. effects. We assume this parameter is equal 3 based on Simonian's (2011) model. We incorporate traditional VaR and COL together in the traditional LaVaR model. The model could involve exogenous liquidity risk and traditional VaR. Thus, the LaVaR formula is expressed as Model 7 (Bangia, 1999, 2001; Shen et al., 2002; Simonian, 2011; Si & Fan, 2012; Tsai & Li, 2015; Tsai & Wu, 2016)

$$\mathrm{LaVaR} = \mathrm{VaR} + \mathrm{COL}, \quad \text{(Model 7)}$$

$$\mathrm{LaVaR} = P_t * \left[\left(1 - \exp\left(-1.96 * \eta_{r,t} * \sigma_{r,t}\right)\right) + 0.5 * \left(\mu_{s1,t} + a * \sigma_{s1,t}\right)\right].$$

3.3 Sensitized Endogenous Liquidity by Hellinger Distance Measure

Following Bogachev (2007) and Simonian (2011) research, we apply Hellinger distance measure to calculate the difference between traditional LaVaR and LaVaR adjusted by endogenous liquidity risk. The Hellinger distance is one of a family of "f-divergences", which can be used to estimate the distance in probability measures. The probability of this measure is a percentage of trade size and it always falls within a range of $[0, 1]$. The Hellinger distance $H(P||Q)$ is as equivalent to the percentage of market trade size (d_H), which is between 0% and 100%. The Hellinger distance can be expressed as Model 8:

$$H(P||Q) = d_H = \sqrt{1 - \int \sqrt{f(x)g(x)}dx}, \quad \text{(Model 8)}$$

where Model8 isthe one-day holding horizon of Hellinger distance. f(x) and g(x) are two probability measures of the state space Ω, which could be used to measure the difference between traditional LaVaR and endogenous adjusted LaVaR at given trade size percentages.

However, when the probabilistic allocation is assumed to be absolutely continuous with a σ-finite dominating measure and with the normal probability distributions $N(\mu_{s1,t}, \sigma_{s1,t})$ and $N(\mu_{s2,t}, \sigma_{s2,t})$, the Hellinger distance (d_H)can be refined as Model 9: (Lagarias et al., 1998)

$$d_H = 1 - \sqrt{\frac{2\sigma_{s1,t}\sigma_{s2,t}}{\sigma_{s1,t}^2 + \sigma_{s2,t}^2} \exp\left(-\frac{1}{4}\frac{\left(\mu_{s1,t} - \mu_{s2,t}\right)^2}{\sigma_{s1,t}^2 + \sigma_{s2,t}^2}\right)}, \quad \text{(Model 9)}$$

where Model 9 isthe equation estimated as one-day holding horizon of Hellinger distance. $\mu_{s1,t}$ and $\sigma_{s1,t}$ are the mean spread and Std. Dev. derived by Model 4, which are parameters of probability distributions under exogenous liquidity risk (i.e., traditional LaVaR). $\mu_{s2,t}$ and $\sigma_{s2,t}$ are respectively the new mean spread and Std. Dev. estimated by Model 9 solved by Nelder-Mead simplex algorithm in the Mathematica 10.0 program, which are parameters for probability distributions incorporating endogenous liquidity risk by the Hellinger distance measure at market trade sizes (i.e., d_H is 1%, 5% and 6%). To include trade size in the model, we treat the trade size percentage as a proxy variable to rectify LaVaR. While previous studies assumed the percentage was only 1% in Simonian (2011), the present study expands this assumption to 5% and 6%, where 6% is the maximum value based on the actual trading percentage, and 5% is average market trading percentage.[5]The parameters $\mu_{s2,t}$ and $\sigma_{s2,t}$ are solved using the Nelder-Mead simplex algorithm in the Mathematica 10.0 program.

Finally, we plug$\mu_{s2,t}$ and $\sigma_{s2,t}$ back into Model 10 to recalculate COL, and use the new COL and traditional VaR to recalculate the new LaVaR asModel 11. Models 10 and 11 are expressed as follows: (Simonian, 2001)

$$\mathrm{COL} = \frac{P_t}{2} * (\mu_{s2,t} + a * \sigma_{s2,t}), \quad \text{(Model 10)}$$

$$\mathrm{LaVaR} = \mathrm{VaR} + \mathrm{COL}, \quad \text{(Model 11)}$$

$$\mathrm{LaVaR} = P_t * \left[\left(1 - \exp\left(-1.96 * \eta_{r,t} * \sigma_{r,t}\right)\right) + 0.5 * (\mu_{s2,t} + a * \sigma_{s2,t})\right].$$

3.4 The Back-testing

Typically, traditional VaR and LaVaR are used to estimate a day-to-day loss at specified left-hand critical value of the portfolio's potential loss distribution. The recommended back-testing guideline proposed by the BCBS (1996) is to evaluate a 5% VaR model over a 12-month test period 250 trading days. (Chen et al., 2012) So, we assume the one-tailed probability is at 5% and the confidence level is at 95% to test the consistency between the practical violation rates and theoretical proportion of failures. For recording and accumulating the day-to-day proportion of failures (POF), we refer to Kupiec's (1995) opinion and denoted the random variable "n" is the number of time for the whole empirical period, and record the consistency between daily ex-post losses and their ex-ante VaR, i.e., the consistency between the practical violation rates and proportion of theoretical failures. When the accumulated number of failures (i.e., ex-post loss is higher than ex-ante VaR) on given period are $\hat{\alpha}$, and the POF recorded is $\hat{p}$, the PDF of POF is a binomial distribution and expressed as following Model 12:

$$\mathrm{Prob}(\hat{p}, \hat{\alpha}) = \hat{p}^{\hat{\alpha}}(1 - \hat{p})^{n-\hat{\alpha}}, \quad \text{Model 12}$$

By the likelihood ratio ($LR_{\mathbf{POF}}$) unconditional coverage (UC) test, we examine the hypothesis $\hat{p}=p_0$ that the practical violation rate is equal to theoretical proportion of failures for an accurate VaR forecasting method. The LR test statisticsexpressed as following. (see Kupiec's (1995) and Gerlach et al. (2016) POF UC test)

$$LR_{POF} = \chi^2 - 2\mathrm{Log}(p_0^{\alpha_0}(1 - p_0)^{n-\alpha_0}) + 2\mathrm{Log}(\hat{p}^{\hat{\alpha}}(1 - \hat{p})^{n-\hat{\alpha}}), \quad \text{Model 13}$$

where Model 13 is LR statistics for testing the consistency between daily ex-post losses and ex-ante VaR. $\hat{\alpha}$ and $\hat{p}$ are number of practical failures and proportion of failures,α_0 and p_0 are number of theoretical failures and theoretical proportion of failures.

4. Empirical Results

4.1 Descriptive Statistics

The descriptive statistics results of 12 qualified SSFs (i.e., CEF, CJF, CKF, CLF, CMF, CNF, DEF, DNF, DOF, DPF, LOF and LRF) are shown in Table 3, including average and Std. Dev. values of return and bid-ask mean spread, each providing 609 daily data samples.The empirical results indicate that CEF has the highest Std. Dev. of return and mean spread, while CJF has the lowest Std. Dev. of return and mean spread. Thus, CEF would have higher traditional VaR and LaVaR, while CEF would have lower traditional VaR and LaVaR. We refer to the table 2 shown, the underlying stock of CEJ is Fubon Financial Holding Co., which ranked second in average asset, and with the first of income, EPS and average price. The underlying stock of CJF is Hua Nan Financial Holdings Co., which ranked sixth in average asset, income and average price, rank seventh of EPS.

5We add the trade size percentages at 1%, 5% and 6% as d_H to broaden the scope of consideration. The absolute value of $d_H$is between 0% and 100% and assumed to be equivalent to the trade size percentage. (Bogachev, 2007 and Simonian, 2011)

Table 3: SSFs Return and Mean Spread Descriptive Statistics

Return	CEF	CJF	CKF	CLF	CMF	CNF
Observations	609	609	609	609	609	609
Mean	0.0714	0.0006	0.0006	0.0001	0.0001	0.0002
Std. Dev.	0.052	0.007	0.015	0.011	0.011	0.013
Return	DEF	DNF	DOF	DPF	LOF	LRF
Observations	609	609	609	609	609	609
Mean	-0.0002	0.0002	-0.0001	-0.0002	-0.0002	0.0003
Std. Dev.	0.013	0.013	0.015	0.010	0.008	0.014
Mean Spread	CEF	CJF	CKF	CLF	CMF	CNF
Observations	609	609	609	609	609	609
Mean	0.0056	0.0013	0.0010	0.0024	0.0024	0.0017
Std. Dev.	0.038	0.001	0.001	0.002	0.001	0.002
Mean Spread	DEF	DNF	DOF	DPF	LOF	LRF
Observations	609	609	609	609	609	609
Mean	0.0024	0.0038	0.0022	0.0023	0.0035	0.0031
Std. Dev.	0.002	0.008	0.002	0.019	0.021	0.003

Note:CEF, CJF, CKF, CLF, CMF, CNF, DEF, DNF, DOF, DPF, LOF and LRF are 12 SSFs listed on TAIFEX. The description of ticker symbol, underlying stock and financial statement are described in table 2.

To observe the return descriptive statistics of all futures, the range of means od return are between -0.0002 and 0.0714, and the Std. Dev. are between 0.007and 0.052. CEF has the highest mean and Std. Dev. of return, while LOF has the lowest average return, and CJF has the lowest Std. Dev. of return.

Furthermore, we observe mean spread descriptive statistics of all futures in advance, the range of means spread are between 0.0010 and 0.0056, the Std. Dev. are between 0.001 and 0.038. CEF also has the highest mean and standard deviation of mean spread, while CKF has the lowest average mean spread, CJF, CKF and CMF have lowest Std. Dev. of mean spread.

4.2 Empirical Result on Traditional VaR

We use return data as a proxy variable to measure traditional market risk and calculate the traditional VaR. We first built a Bollerslev (1986) GARCH regression model for all SSFs as Model 3. The coefficient β of all SSFs is estimated between 0.713 and 0.959 as presented in Table 4. The empirical GARCH results also indicate that the prior period volatility return $\sigma^2_{r,t-1}$ of all SSFs would significantly influence the current volatility $\sigma^2_{r,t}$ with quasi maximum likelihood estimation (QMLE) (Berndt et al., 1974). The other evaluation of GARCH estimated results, including QMLE, and Akaike information criterion (AIC) (Akaile, 1974) are also shown in Table 4. The empirical results show the return data follows the non-N.D., stationary, and heteroscedasticity. (Jarque & Bera, 1987; Dickey & Fuller, 1979; Engle, 1982).

Table 4: Empirical Results for Single Stock Futures Using the Traditional VaR Model

Futures		CEF	CJF	CKF	CLF	CMF	CNF
Raw data test[a]	Normal dist. test	1114.6*	890.0*	985.3*	1308.3*	844.4*	434.4*
	Unit root test	-26.016*	-29.213*	-28.029*	-29.128*	-29.093*	-26.815*
	Heteroscedasticity test	5.205*	4.902*	6.505*	5.345*	6.272*	5.444*
GARCH[b]	α	0.001	0.001	0.001	0.001	0.001	0.001
	β	0.880*	0.929*	0.887*	0.959*	0.713*	0.784*
	ρ	0.119	0.828	0.035	0.028	0.064	0.141
	QMLE	1361.7	1502.6	1318.3	1332.9	1370.9	1353.4
	AIC	-5.719	-6.314	-5.535	-5.596	-5.758	-5.684
Average VaR[c]		0.173	0.021	0.074	0.110	0.061	0.061
Failure rate[d]		0.098*	0.130*	0.118*	0.090*	0.082*	0.108*
Futures		DEF	DNF	DOF	DPF	LOF	LRF

Futures		CEF	CJF	CKF	CLF	CMF	CNF
Raw data test[a]	Normal dist. test	513.2*	474.0*	989.6*	835.7*	458.7*	849.6*
	Unit root test	-27.341*	-28.731*	-28.505*	-28.278*	-26.443*	-27.160*
	Heteroscedasticity test	4.762*	5.012*	5.623*	4.692*	6.392*	5.001*
GARCH[b]	α	0.001	0.001	0.001	0.001	0.001	0.001
	β	0.881*	0.872*	0.931*	0.747*	0.866*	0.919*
	ρ	-0.052*	0.105	0.330	0.048	0.516	0.053
	QMLE	1332.2	1403.6	1327.9	1340.0	1299.9	1313.7
	AIC	-5.745	-6.340	-5.562	-5.623	-5.784	-5.710
Average VaR[c]		0.066	0.124	0.122	0.077	0.067	0.076
Failure rate[d]		0.090*	0.110*	0.082*	0.048	0.044	0.108*

Note: * reject H_0 at α=0.05.

a. The hypothesis of the autoregressive model is H_0^1: The autoregressive model is "N.D.", H_0^2: The time series is non-stationary and H_0^3: The autoregressive model is not heteroscedastic.
b. The hypothesis of the GARCH model is H_0: α=0, β=0 and γ=0.
c. Traditional VaR is defined asModel 5.
d. The hypothesis of Kupiec's back-testing is H_0: $\hat{p}=p_0$.

Furthermore, we plug $\mu_{r,t}$, $\sigma_{r,t}$and P_t into Model 5 to calculate the traditional VaR for each trade date, and adjust the non-N.D. by correction factor parameter $\kappa_{r,t}$. The traditional VaR of 12 SSFs are between 0.021 and 0.173. CEF has the highest VaR, while CJF has the lowest VaR.[6] We then apply Kupiec's (1995) POF test by the null hypothesis H_0: $\hat{p}=p_0$, which assumed the practical failure rates ($\hat{\alpha}$) are consistent with their theoretical failure rates (α_0). By using $LR_{\mathbf{POF}}$ unconditional coverage test, we examine the consistency of practical violation rates and proportion of theoretical failures at a 5% one-tailed probability and 95% confidence level.[7] Thus, following Models 12and 13, the empirical results shown in Table 4 indicate that the practical failure rates are between 0.044 and 0.133, and only DPF and LOF significantly accept the null hypothesis. Thus, it seems that considering only the traditional VaR would misjudge the risk, and another risk must be integrated to adjust for this condition, e.g., liquidity risk.

4.3 Empirical Result on Traditional LaVaR

We use the bid-ask mean spread as a proxy variable to measure the exogenous liquidity risk and calculate the traditional LaVaR. We first built a Bollerslev (1986) GARCH regression model forall SSFs as Model 4. The coefficient θ is estimated between 0.744 and 0.940 as presented in Table 5. The empirical GARCH results also indicate that the prior period volatility of mean spread $\sigma^2_{s1,t-1}$ would significantly influence the current volatility $\sigma^2_{s1,t}$ with QMLE (Berndt et al., 1974). The other evaluation of GARCH estimated results, including QMLE and AIC, are also shown in Table 5. The empirical results show the mean spread data follows the non-N.D., stationary, and heteroscedasticity.(Jarque & Bera, 1987; Dickey & Fuller, 1979; Engle, 1982)

Furthermore, we plug $\mu_{s1,t}$, $\sigma^2_{s1,t}$ and P_t into Model 6 to calculate COL for each trade date by scaling-adjusted parameters assumed at 3 as Simonian (2011), which modify the estimated bias due to the combined non-N.D. effects. Therefore, we sum up COL and traditional VaR, i.e., traditional LaVaR as Model 7. The range of the average COL for all SSFs are between 0.001 and 0.014. The range of the average LaVaR for all SSFs are between 0.022 and 0.187. CEF has the highest LaVaR, while CJF has the lowest LaVaR.[8]

6 Refer to the table 2 shown, the underlying stock of CEJ is Fubon Financial Holding Co., that ranked second in average asset, and with the first of income, EPS and average price. The underlying stock of CJF is Hua Nan Financial Holdings Co., that ranked sixth in average asset, income and average price, rank seventh of EPS.

7When the ex-post loss is higher than ex-ante VaR in a given period, the number of failures $\hat{\alpha}$ are accumulated and the POF is also recorded. By the likelihood ratio of unconditional coverage test, we examine the hypothesis H_0: $\hat{p}=p_0$ at one-tailed probability and confidence level are respectively 5% and 95%. (Kupiec, 1995; Chan et al., 2012; Gerlach et al., 2016)

8Refer to footnote6.

Moreover, we apply Kupiec's (1995) POF test and build the null hypothesis H_0: $\hat{p}=p_0$ assumed the practical failure rates ($\hat{\alpha}$) are consistent with their theoretical failure rates (α_0). By LR_{POF} unconditional coverage test, we examine the consistency of practical violation rates and proportion of theoretical failures at a 5% one-tailed probability and 95% confidence level.[9] Thus, following Models 12 and 13, the empirical results shown in Table 5 indicate that the practical failure rates are between 0.042 and 0.059. According to the results of all likelihood ratio tests shown that 12 SSFs significantly accept the null hypothesis. It seems that, by incorporating exogenous liquidity risk and traditional VaR together would provide a more appropriate risk level.

Table 5: Empirical Results for Single Stock Futures Using the Traditional LaVaR Model

Futures		CEF	CJF	CKF	CLF	CMF	CNF
Raw data test[a]	Normal dist. test	862.5*	872.3*	426.1*	316.1*	989.7*	564.8*
	Unit root test	-13.172*	-9.227*	-12.121*	-12.134*	-8.334*	-9.304*
	Heteroscedasticity test	4.705*	5.364*	4.082*	4.927*	5.179*	6.103*
GARCH[b]	π	0.001	0.001	0.001	0.001	0.001	0.001
	θ	0.940*	0.838*	0.895*	0.884*	0.817*	0.744*
	τ	0.054	0.148	0.035	0.106	0.173	0.235
	QMLE	1457.1	473.6	1639.0	1220.1	1240.6	1145.6
	AIC	-6.148	-4.190	-6.918	-5.146	-5.233	-4.831
Scaling-adjusted parameters a=3: [c]							
Average COL		0.014	0.001	0.002	0.005	0.005	0.004
Average LaVaR		0.187	0.022	0.076	0.115	0.066	0.065
Failure rate		0.057	0.059	0.055	0.052	0.055	0.059
Futures		DEF	DNF	DOF	DPF	LOF	LRF
Raw data test[a]	Normal dist. test	470.3*	725.2*	804.5*	1019.5*	141.8*	154.4*
	Unit root test	-9.815*	-14.968*	-9.040*	-11.542*	-10.791*	-14.114*
	Heteroscedasticity test	4.290*	4.901*	6.399*	5.173*	5.550*	4.937*
GARCH[b]	π	0.001	0.001	0.001	0.001	0.001	0.001
	θ	0.762*	0.872*	0.837*	0.901*	0.783*	0.752*
	τ	0.218	0.105	0.183	-0.002	0.288	0.125
	QMLE	1205.3	1411.1	1290.6	1309.1	1300.9	1267.8
	AIC	-6.122	-1.964	-6.892	-5.120	-5.206	-4.805
Scaling-adjusted parameters a=3: [c]							
Average COL		0.005	0.013	0.004	0.004	0.008	0.006
Average LaVaR		0.071	0.137	0.126	0.081	0.075	0.082
Failure rate		0.055	0.049	0.042	0.048	0.044	0.056

Note: * reject H_0 at α=0.05.

a. and b. please see the explanations in Table 4.

c. COL and LaVaR is defined asModels 6 and 7. The hypothesis of Kupiec's back-testing is H_0: $\hat{p}=p_0$.

4.4 Empirical Result of Hellinger Distance Measure on LaVaR

As described in section 4.3, when the scaling-adjusted parameter of traditional LaVaR is set at 3, 12 SSFs significantly pass the back-testing hypothesis. Thus, we use the traditional LaVaR with a scaling-adjusted parameter of 3 as a based model to sensitize for the endogenous liquidity risk effect. For measuring the endogenous liquidity risk, we use the Hellinger distance measure $d_H$as a series assuming different trade sizes.Following the Simonian's (2011) research, and the average and maximum trade size percentages of 12 SSFs in TAIFEX are respectively 5% and 6%, we consider $d_H$at 1%, 5%, and 6% as proxy variables to measure the incremental adjustment of endogenous liquidity risk on traditional LaVaR. By combining the probability measure of the Hellinger distance measure characteristics considered in Bogachev's (2007) and Simonian's (2011) research, we plug d_H, $\mu_{s1,t}$ and $\sigma_{s1,t}$ together into Model 9 to estimate $\mu_{s2,t}$ and $\sigma_{s2,t}$, which provide a sensitivity analysis for

9Refer to footnote7.

the endogenous liquidity risk. Table 6 shows the empirical results obtained from the Nelder-Mead simplex algorithm in Mathematica 10.0 by Model 9.

By plugging $\mu_{s2,t}$ and $\sigma_{s2,t}$ estimates into the COL calculation as in Model 10, we recalculate the new average COL for all SSFs, and incorporating COL and traditional VaR as Model 11, we recalculate the new LaVaR. Table 6 shows empirical results that CEF has the highest LaVaR, and CJF has the lowest LaVaR of all 12 SSFs.[10]On the whole, the results indicate that $\mu_{s2,t}$ and $\sigma_{s2,t}$ increase with the total market trade size percentage (i.e., 1%, 5%, and 6%).It also indicates that integrating endogenous liquidity risk, new COL and LaVaR would increase with trade size, and that are larger than the LaVaR only exogenous liquidity risk considered. The result of new COL and LaVaR are as followings:

(a) At 1% of market trade size, the COL of all SSFs are between 0.0022 and 0.0710. We then sum up the traditional VaR and the COL. The LaVaR of all SSFs is between 0.0232 and 0.2440.
(b) At 5% of market trade size, the COL of all SSFs is between 0.0337 and 0.2252, and the total LaVaR is between 0.0547 and 0.3982.
(c) At 6% of market trade size, the COL of all SSFs is between 0.0495 and 0.3797, and the LaVaR is between 0.0705 and 0.5527.

To evaluate the accuracy of the new LaVaR model, we apply Kupiec's (1995) POF test and build the null hypothesis H_0: $\hat{p}=p_0$ assumed the practical failure rates ($\hat{\alpha}$) are consistent with their theoretical failure rates (α_0). By $LR_{\mathbf{POF}}$ unconditional coverage test, we examine the consistency of practical violation rates and proportion of theoretical failures at a 5% one-tailed probability and 95% confidence level.[11] Thus, for counting the number of failure rates and test the consistency, we follow the Models 12 and 13. The practical failure rates shown in Table 6 are between 0.042 and 0.059, and according to likelihood ratio test, 12 SSFs are all significantly accept the null hypothesis. It seems that, by incorporating endogenous liquidity risk would provide an appropriate risk level.

Table 6: Empirical Results for SSFs Sensitized to Trade Size in LaVaR

Futures			CEF	CJF	CKF	CLF	CMF	CNF
Trade Size Percentage[a, b]	1%	Average $\mu_{s2,t}$	0.0141	0.0019	0.0029	0.0029	0.0031	0.0028
		Average $\sigma_{s2,t}$	0.0100	0.0001	0.0010	0.0029	0.0028	0.0001
		Average COL	0.0710	0.0022	0.0059	0.0116	0.0115	0.0031
		Average LaVaR	0.2440	0.0232	0.0799	0.1216	0.0725	0.0641
		Failure rate	0.057	0.059	0.055	0.052	0.055	0.059
	5%	Average $\mu_{s2,t}$	0.0472	0.0250	0.0414	0.0420	0.0427	0.0046
		Average $\sigma_{s2,t}$	0.0460	0.0029	0.0046	0.0047	0.0047	0.0419
		Average COL	0.2252	0.0337	0.0552	0.0561	0.0568	0.1303
		Average LaVaR	0.3982	0.0547	0.1292	0.1661	0.1178	0.1913
		Failure rate	0.057	0.059	0.055	0.052	0.055	0.059
	6%	Average $\mu_{s2,t}$	0.1034	0.0378	0.0541	0.0636	0.0652	0.0835
		Average $\sigma_{s2,t}$	0.0921	0.0039	0.0072	0.0093	0.0092	0.0093
		Average COL	0.3797	0.0495	0.0757	0.0915	0.0928	0.1114
		Average LaVaR	0.5527	0.0705	0.1497	0.2015	0.1538	0.1724
		Failure rate	0.057	0.059	0.055	0.052	0.055	0.059

10Refer to footnote6.
11Refer to footnote7.

Futures			DEF	DNF	DOF	DPF	LOF	LRF
Trade Size Percentage[a, b]	1%	Average $\mu_{s2,t}$	0.0029	0.0040	0.0028	0.0029	0.0089	0.0038
		Average $\sigma_{s2,t}$	0.0029	0.006	0.0029	0.0034	0.0007	0.0330
		Average COL	0.0116	0.0199	0.0115	0.0131	0.0040	0.1028
		Average LaVaR	0.0776	0.1439	0.1335	0.0901	0.0710	0.1788
		Failure rate	0.055	0.049	0.042	0.048	0.044	0.056
	5%	Average $\mu_{s2,t}$	0.0422	0.0166	0.0039	0.0420	0.0494	0.0421
		Average $\sigma_{s2,t}$	0.0051	0.0072	0.0047	0.0048	0.0042	0.0046
		Average COL	0.0575	0.0382	0.0180	0.0564	0.0621	0.0559
		Average LaVaR	0.1235	0.1622	0.1400	0.1334	0.1290	0.1319
		Failure rate	0.055	0.049	0.042	0.048	0.044	0.056
	6%	Average $\mu_{s2,t}$	0.0639	0.0203	0.0251	0.0644	0.0742	0.0737
		Average $\sigma_{s2,t}$	0.0093	0.0086	0.0074	0.0092	0.0095	0.0084
		Average COL	0.0918	0.0461	0.0473	0.0920	0.1027	0.0989
		Average LaVaR	0.1578	0.1701	0.1693	0.1690	0.1697	0.1749
		Failure rate	0.055	0.049	0.042	0.048	0.044	0.056

Note: * reject H_0 at α=0.05.

a. d_H is 0% as Bangia et al. (1999, 2001) model; 1% is as Simonian (2011) model. 6% and 5% are the maximum and average value based on the actual trading percentage. The new $\mu_{s2,t}$ and $\sigma_{s2,t}$ are estimated by Model 9, i.e., d_H.

b. New COL and LaVaR is defined as Models 10 and 11. The hypothesis of Kupiec's back-testing is H_0: $\hat{p}=p_0$.

5. Conclusions

This paper uses the empirical model following Bangia et al. (1999, 2001) in dividing liquidity risk into exogenous and endogenous types. However, Bangia et al. (1999, 2001) exclusively emphasize COL and exogenous liquidity risk calculations, without accounting for endogenous liquidity risk. We use Simonian's (2001) empirical concept incorporating the Hellinger distance measure to calculate the effect of percentage of trade size and modify the endogenous liquidity risk for the exogenous LaVaR. Simonian (2011) only sensitizes the liquidity effect at the trade size 1% assumed, thus we add trade size percentages at 1%, 5% and 6% to broaden the scope of consideration. By sensitizing these three different trade size effects to value the endogenous liquidity effect on traditional LaVaR, we recalculate new $\mu_{s2,t}$ and $\sigma_{s2,t}$, then revalue the new COL, and plug COL to measure the new LaVaR. Thus, the major contribution of this paper is to incorporate the endogenous liquidity risk effect and re-estimate the exogenous LaVaR using the Hellinger distance measure.

By combining the probability measure of the Hellinger distance characteristics, applying sensitivity analysis to involve the endogenous liquidity risk, and adjusting the traditional LaVaR considered at the exogenous liquidity risk, the major research findings are as follows:

(a) The empirical results of traditional VaR are between 0.021 and 0.173.
(b) The empirical results of the traditional LaVaR between 0.022 and 0.187.
(c) The empirical results of LaVaR adjusted by endogenous liquidity risk are between 0.0232 and 0.5527.

On the whole, CEF produces the highest LaVaR, while CJF has the lowest LaVaR for all SSFs. Applying Kupiec's (1995) POF test, the practical failure rates of all SSFs are largely consistent with their theoretical failure rates in traditional LaVaR model and new LaVaR adjusted by endogenous liquidity risk.

References

[1] Al Janabi, M.A.M., 2011a. "A generalized theoretical modeling approach for the assessment of economic capital under asset market liquidity risk constraints",*The Service Industries Journal*31. pp. 2193-2221.

[2] Al Janabi, M.A.M., 2011b. "Dynamic equity asset allocation with liquidity-adjusted market risk

criterion: appraisal of efficient and coherent portfolios", *Journal of Asset Management*12, pp. 378-394.

[3] Al Janabi, M.A.M., 2013. "Optimal and coherent economic-capital structures: evidence from long and short-sales trading positions under illiquid market perspectives",*Annals of Operations Research*205, pp.109-139.

[4] Akaike, H.1974."A new look at the statistical model identification",*IEEE Transactions on Automatic Control*19, pp.716-723.

[5] Aubier, A. and E. Le., Saout, 2002. "La nature du risque de liquidité: existe-t-il un risque systématique de liquidité?", *Cahier de recherche du CEREG*. N° 9611.

[6] Aubier, A., 2006. "Les mouvements extrêmes de liquidité", *Banques et Marchés* 84, pp.20–29.

[7] Bangia, A.,F.X., Diebold, T., Schuermann, and J.D., Strounghair, 1999."Modeling liquidity risk with implications for traditional market risk measurement and management", *Financial Institutions Center at the Wharton School Working Paper*, 1-18.

[8] Bangia, A., F.X., Diebold, T., Schuermann, and J.D., Strounghair, 2001."Modeling liquidity risk, with implications for traditional market risk measurement and management. In Risk Management: The State of the Art, Edited by Figlewski, S. and Levich, R. M.", 3-13,*The New York University Salomon Center Series on Financial Markets and Institutions 8*, Springer US.

[9] Berndt, E.R., B.H., Hall, R.E., Hall, and J.A., Hausman, 1974."Estimation and inference in nonlinear structural models",*Annals of Economic and Social Measurement*4, pp.653-665.

[10] Bogachev, V., 2007."*Measure Theory*", Spring-Verlag, Berlin.

[11] Bollerslev, T., 1986. "Generalized autoregressive conditional heteroscedasticity", *Journal of Econometric*31, pp.307-327.

[12] Black, F., 1971a. "Towards a fully automated exchange I",*Financial Analysts Journal*27, pp.28-44.

[13] Black, F., 1971b. "Towards a fully automated exchange II",*Financial Analysts Journal*27, pp.24-87.

[14] Chen, C.W.S., R., Gerlach,E.M.H.,, Lin, and W.C.W., Lee, 2012. "Bayesian forecasting for financial risk management, pre and post the global financial crisis", *Journal of Forecasting*31, pp.661-687.

[15] Chang, Y.P., J.X., Lin, and C.T., Yu, 2016. "Calculating Value-at-Risk using the granularity adjustment method in the portfolio credit risk model with random loss given default", *Journal of Economics and Management* 12, pp.1-20.

[16] Cosandey, D., 2001."Adjusting value-at-risk for market liquidity",*Risk*14, pp.115-118.

[17] Demsetz, H., 1968. "The cost of transacting",*Quarterly Journal of Economics*82, pp.33-53.

[18] Dickey, D.A.and W.A, Fuller, 1979."Distribution of the estimators for autoregressive Time series with a unit root",*Journal of the American Statistical Association*74, pp.427–431.

[19] Engle, R.F., 1982."Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation",*Econometrica*50, pp.987-1007.

[20] Gerlach, R., C.W.S., Chen, and E.M.H., Lin, 2016."Bayesian Assessment of Dynamic Quantile Forecasts", *Journal of Forecasting*35, pp.669-764.

[21] Glosten, L.R. and L.E., Harris, 1988. "Estimating the components of the bid/ask spread",*Journal of Financial Economics* 21, pp.123-142.

[22] Häberle, R. and P.G., Persson, 2000. "Incorporating market liquidity constraints in value-at-risk",*Banques & Marchés*44, pp.14-19.

[23] Jarque, C.M. andA.K., Bera, 1987. "A test for normality of observations and regression residuals",*International Statistical Review* 55, pp.163-172.

[24] Jarrow, R.A. and A., Subramanian, 1997. "Mopping up liquidity",*Risk*10, pp.170-173.

[25] Jarrow, R.A., P.F, Philip, and A., Roch,2012. "Liquidity-based model for asset price bubbles",*Quantitative Finance* 12, pp.1339-1349.

[26] Jorion, P., 2007."*Value at Risk"*, New York: McGraw-Hill.

[27] Kupiec, P., 1995. “Techniques for verifying the accuracy of risk measurement models”,*Journal of Derivatives* 3, pp.73-84.

[28] Kyle, A.S., 1985. “Continuous auctions and insider trading”,*Econometrica*53, pp.1315-1335.

[29] Lagarias, J.C., J.A., Reeds, M.H., Wright, and P.E., Wright, 1998.“Convergence properties of the Nelder-Mead simplex method in low dimensions”, *SIAM Journal of Optimization* 9, pp.112–47.

[30] Le Saout, E., 2002. “Incorporating liquidity risk in VaR models”, mimeo, Paris I University – CREFIB & CREREG.

[31] Morgan, J.P., 1996. “RiskMetrics Technical Documents”, New York.

[32] Ourir A. and W., Snoussi, 2012.“Markets liquidity risk under extremal dependence: analysis with VaRs methods”,*Economic Modelling*29, pp.1830-1836.

[33] Shen, D.B., J.N., Yang, and X.Z., Huang, 2002.“Measurement of liquidity risk”, Money *Watching and Credit Rating* 37, pp.115-127.

[34] Si, W.M. and X.I., Fan, 2012.”The calculation on liquidity adjusted VaR of Taiwan stock”,*Money Watching and Credit Rating*95, pp.78-91.

[35] Simonian, J., 2011.“Liquidity on the outside from the inside”,*Applied Economics Letters*18, pp.1591-1593.

[36] Stoll, H.R., 2000.“Friction”,*The Journal of Finance*55, pp.1479-1514.

[37] Subramanian, A. and R.A., Jarrow, 2001.“The liquidity discount”,*Mathematical Finance*11, pp.447-474.

[38] Tsai, C.C. and T.S., Li, 2015.“A revaluation study of liquidity-adjusted VaR on TAIFEX index futures”,*Journal of Futures and Options*8, pp.1-40.

[39] Tsai, C.C. and Y.Z., Wu, 2016.“Research of Liquidity-adjusted Value at Risk in Taiwan Stock Futures”,*Journal of Risk Management*17, pp.59-87.

[40] Zhan, C. and X.Y., Hun, 2001.“The review of liquidity measure”,*Proceedings of the National Science Council*11, pp.205-221.