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Description of a Diffusion Price Model with Uniform Constant Jump Rate

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Abstract

There is a considerable interest in stochastic analogs of classical difference and differential equations describing phenomena in theoretical models involving economic structure. In this paper a description of diffusion price model with Uniform constant jump process using a solution of stochastic differential equation is considered. The Moments of such price model is studied. More specifically, the mean and the variance as well as the sample path of such a process are determined.

Keywords: Stochastic differential equation, Diffusion price process, Uniform jump process
JEL Classification: E31

1. Literature Review

This paper shows how decision makers' concerns about model specification can affect prices and quantities in a dynamic economy. The important reason for using this new approach of stochastic process in price models is this kind of models is not widely used in various economic models.

Hand (2001) have developed methods using statistical tools such as logisting regression and naïve Bayes as well as neural networks for assessing performance of the models to the consumer credit risk.

Veronesi (1999) studies a permanent income model with a risckless linear technology. Dividends are modeled as an additional consumption endow cent. Hidden information was introduced into asset pricing models by Detemple (1986), who considers a production economy with Gaussian unobserved variables.

David (1997) studies a model in which production is linear in the capital stocks with technology shocks that have hidden growth rates.

During this past decade there has been increasing effort to describe various facts of dynamic economic interactions with the help of stochastic differential processes. Thus stochastic differential processes provide a mechanism to incorporate the influences associated with randomness, uncertainties, and risk factors operating with respect to various economic units (stock prices, labor force, technology variables, etc.)

Numerous researchers have worked on studying various economic units from different points of view. For example, Aase and Guttrop (1987) studied the role of security prices allocative in capital market, they present stochastic models for the relative security prices and show how to estimate these random processes based on historical price data. The models they suggest may have continuous components as well as discrete jumps at random time points. Also, one of the classical applications is

Black and Scholes (1973). New references include Harrison and Pliska (1981) and Aase (1984). Whereas the first two works only study processes with continuous sample paths, the other two allow for jumps in the paths as well. In other words, the processes have sample paths that are continuous from the right and have left hand limits (in fact, these processes are semi-martingales; for general theory of semi-martingales, see e.g. Kabanov et al., 1979 sec. 2).

Many other authors have studied this problem from different points of view, such as Stein and Stein (1991), Tauchen and Pitts (1983), Schwert (1990), Duffie and Singleton (1993), McGrattan (1996), Callen and Chang (1999), Karmeshu and Goswami (2001), etc.

In this paper, we present a new diffusion price model with Uniform constant jump process using a solution of stochastic differential equation. The Moments of such price model is studied. More specifically, we consider the solution of the stochastic differential equation (SDE), and study the mean, variance as well as the sample path of such a process are determined.

2. Method

Consider a birth and death diffusion price model in which the diffusion coefficient a and the drift coefficient b are both proportional to the price S_t at time t. The diffusion process is assumed to be interrupted by upward and downward jumps occurring at a constant rate c and having magnitudes with distribution function $H(\cdot)$. Then $\{S, t \geq 0\}$ is a Markov process with state space $S=[0, \infty)$ and generator $\boldsymbol{g}$, where

$$gf(s) = asf''(s) + bsf'(x) + c\int_{[0,s)}[f(s-y) - f(s)]dH_s(y) \\ + c[f(0) - f(s)][1 - H(s^-)], \tag{1}$$

for all $f \in \boldsymbol{D}(\boldsymbol{g})$, where $\boldsymbol{D}(\boldsymbol{g})$ is the domain of $\boldsymbol{g}$, and $a>0$, $c>0$, $\varepsilon>0$. Let $F(s, t)$ be the distribution function of the process at time t, i.e., $F(s, t) = P(S_t \leq s)$. Let $\varphi(\theta, t)$ be the Laplace transform of S_t, i.e.

$$\varphi(\theta,t)=\int_0^\infty e^{-\theta s}\, dF(s,t) \tag{2}$$

where

$$\Phi(\theta,s)=e^{s} \tag{3}$$

Observing that

$$g\Phi = as\theta^2 e^{-\theta x} - bs\theta e^{-\theta x} + c\int_0 [e^{-\theta(s-y)} - e^{-\theta s}]dH_s(y) \\ + c[1 - e^{-\theta s}][1 - H_s(s^-)], \tag{4}$$

and substituting in the ((see Breiman (1968), P. 327))

$$\frac{d}{dt}\int \Phi dF(s,t) = \int (g\Phi) dF(s,t)$$

gives

$$\frac{d\varphi}{dt} = \int_0^\infty \,[as\theta^2 e^{-\theta s} - bs\theta e^{-\theta s} + c\int_0^{s^-}[e^{-\theta(s-y)} - e^{-\theta s}]dH_s(y) \\ + c[1 - e^{-\theta s}][1 - H_s(s^-)]]dF(s,t). \tag{5}$$

Since in the section 3 we will consider the Uniform jump distribution which is continuous, then in this case s^{-} may be replaced by s, and by performing the Laplace transforms in (5), we find for the constant jump rate that equation(5) may be written as

$$\frac{d\varphi}{dt}=(b\theta-a\theta^2)\frac{d\varphi}{d\theta}-c\varphi(\theta,t)+c\int_0^{\infty}e^{-\theta s}\int_0^{s}e^{\theta y}\,dH_s(y)dF(s,t) \tag{6}$$

3. Population Moments of a Diffusion Price Model with Constant Uniform Jump Rate

In this section we consider the moments $M_n(t)$, n=1,2,... of a diffusion price process with constant jump rate and Uniform jump process. $H_s(y)$ defined by

$$H_s(y)=\begin{cases}0 & , y<0\\ y & , 0\le y<s,\\ 1 & , y\ge 0\end{cases} \tag{7}$$

The equation (6) can be written as

$$\frac{d\varphi}{dt}=(b\theta-a\theta^2)\frac{d\varphi}{d\theta}-c\varphi(\theta,t)+c\int_0^1\varphi(\theta w)dw. \tag{8}$$

Now,

$$\frac{d}{dt}\left(\frac{d\varphi}{d\theta}\right)=(b-2a\theta)\frac{d\varphi}{d\theta}+(b\theta-a\theta^2)\frac{d^2\varphi}{d\theta^2}-c\frac{d\varphi}{d\theta}+c\int_0^1\varphi'(\theta w)dw.$$

By taking $\theta\to 0$, we get

$$\frac{d}{dt}(M_1(t))=(b-\frac{c}{2})M_1(t) \tag{9}$$

Also,

$$\frac{d}{dt}\left(\frac{d^2\varphi}{d\theta^2}\right)=-2a\frac{d\varphi}{d\theta}+2(b-2a\theta)\frac{d^2\varphi}{d\theta^2}-c\frac{d^2\varphi}{d\theta^2}+(b\theta-a\theta^2)\frac{d^3\varphi}{d\theta^3}+c\int_0^1\varphi''(\theta w)dw$$

By taking $\theta\to 0$, we get

$$\frac{d}{dt}(M_2(t))=2aM_1(t)+(2b-\frac{2c}{3})M_2(t). \tag{10}$$

Thus, by taking the higher derivative of φ with respect to θ and by letting θ goes to 0 we obtain the general equations

$$\frac{d}{dt}(M_n(t))=n(n-1)aM_{n-1}(t)+(nb-\frac{nc}{n+1})M_n(t) \tag{11}$$

for all n=1,2,...Equivalently, defining

$$\vec{M}(t)=[M_1(t),M_2(t),M_3(t),\ldots]'$$

these equations can be written in the form

$$\frac{d\vec{M}(t)}{dt} = A\vec{M}(t) \tag{12}$$

where A is given by

$$\begin{bmatrix} b-\frac{c}{2} & 0 & 0 & 0 & \cdots \\ 2a & 2b-\frac{2c}{3} & 0 & 0 & \cdots \\ 0 & 6a & 3b-\frac{3c}{4} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The solution of equation (12), giving the moments of the process $\{S_t\}$ is

$$\vec{M}(t)=e^{At}\vec{M}(0). \tag{13}$$

If $S_0 = s$ then

$$\vec{M}(0) = \begin{bmatrix} s \\ s^2 \\ s^3 \\ \vdots \end{bmatrix}$$

The matrix e^{At} is defined as

$$e^{At}=I+At+\frac{A^2t^2}{2!}+\frac{A^3t^3}{3!}+\ldots$$

This series converges for all t since A is lower triangular. In fact the components $M_n(t)$ of M(t) can be computed recursively from the equations (9) and (11). Thus, in particular,

$$M_1(t)=s\exp\left\{\left(b-\frac{c}{2}\right)t\right\} \tag{14}$$

and

$$M_2(t)=\frac{2as}{\frac{c}{6}-b}\left[\exp\left\{\left(b-\frac{c}{2}\right)t\right\}-\exp\left\{\left(2b-\frac{2c}{3}\right)t\right\}\right]+s^2\exp\left\{\left(2b-\frac{2c}{3}\right)t\right\} \tag{15}$$

Consequently the mean and the variance of the diffusion price model S_t with constant uniform jump rate are given by

$$E(S_t)=s\exp\left\{\left(b-\frac{c}{2}\right)t\right\} \tag{16}$$

and

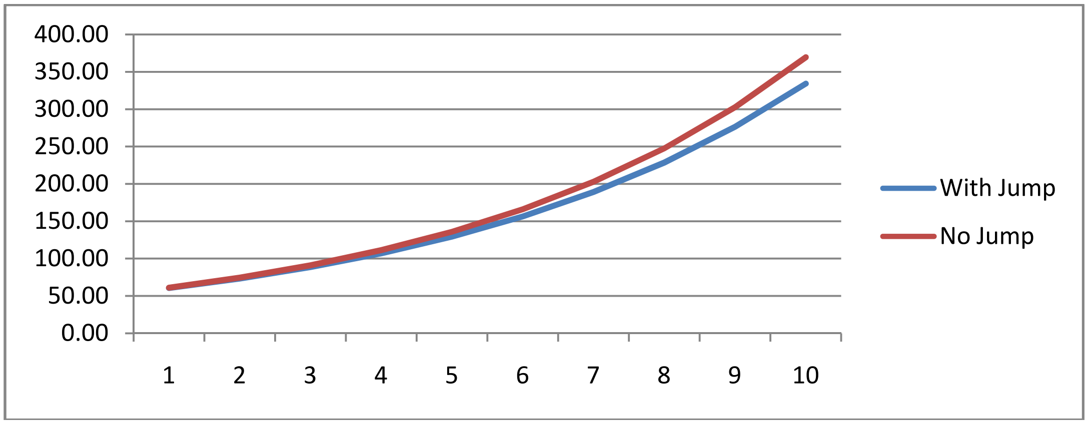
$$\begin{aligned} Var(S_t) &= \frac{2as}{\frac{c}{6}-b}\left[\exp\left\{\left(b-\frac{c}{2}\right)t\right\}-\exp\left\{\left(2b-\frac{2c}{3}\right)t\right\}\right] \\ &\quad + s^2\exp(2bt)\left[\exp\left(-\frac{2c}{3}t\right)-\exp(-ct)\right] \end{aligned} \tag{17}$$

These results usually used in statistical inference problems.

4. Numerical Example

Consider the following example which shows the first moment (mean of prices) using the diffusion price model S_t of some commodity over a period of 10 years starting with initial price $s = 50$. Assume that the model parameters are as follows: $a = 0.1$, $b = 0.2$ and $c = 0.02$ we get:

Figure 1: The Mean of the Diffusion Price Model with Constant Uniform Jump Rate



This graph shows how the jump affect the prices and shows the exponentially trend in the mean of prices over the 10 years period. However should be very unlikely if the parameters are representative of a real price.

5. Conclusions and Extensions

This study provided a methodology for studying the behavior of the prices. More specifically, the study departs from the traditional before - and – after regression techniques and the time series analysis and developed a stochastic model that explicitly accounts for the variations in prices a random environment.

In terms of future research, this methodology could be applied not only in prices but on all aspects of economics problems.

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