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Model with Cycles for Refinancing Individual Lending in Microcredit

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Abstract

This work proposes a new model for an optimal refinancing of individual lending contract in microcredit, in view of strategic default. The aim is to minimize the risk of no repayment related to this contract and to maximize the borrower's welfare allowing him to have a financial and technical autonomy, and to be Self Risk Manager. The considered model permits to compute the most important parameters of the contract.

Keywords: Individual lending, Strategic default, Modeling, Risk of no- repayment, Cycles of refinancing, Self risk manager.

1. Introduction

Since its implementation by Grammen Bank in Bangladesh, founded by Muhamud Yunus Nobel Peace Prize in 2006 Armadariz de Agion (2010), the Microfinance known an expansion thanks to its efficiency for economic and social development in developing countries, including the fight against poverty.

Services offered by the Canadian Institutes of Microfinance (MFI), especially microcredit, are intended for people who do not have real material guarantees that allow them access to formal financial services.

This absence of collateral generates products at higher risk. Today, the major challenge in the MFI is the development of safer products to minimize adverse effects of asymmetric Information (AI) between the MFI and its borrowers, but also by the presence of the default strategy.

The MFI offers two types of contracts, the individual lending and lending group, characterized by several parameters. The most important are the interest and shared responsibility in the collective lending. The latter characteristic is that each borrower declares repay (s) part (s) of its partner (s) failed (s) and the decision of reimbursement depends on the entire group. As for the individual lending, the decision is up to the borrower.

In this work, we are interested in studying the individual lending. Indeed, the understanding and modeling of collective lending requires the study of individual lending.

After signing the contract and the lending, the MFI cannot observe or predict the actions that the borrower can take, for this we will need to use the theory of incentive contracts Ghatak (1999), based on two mechanisms: the incentive and participation.

Bhole and Ogden (2010) suggests that the incentive takes the form of refinancing the project if the borrower fully or partially respects the clauses in his contract , contrary to what has been discussed by Besley and Coate (1995) which propose in their article, that with more penalties and more severity the reimbursement rate is more important. We propose in this work a model which makes less severe sanctions by replacing them by more cash charges which the borrower can support.

The goal is that the borrower has the sustainability of financial and technical autonomy, and the borrower will be Self Risk Manager (SRM). The flow or SRM will allow the borrower to manage optimally lending and project and reduce cash expenses.

In this paper we propose a new contract model for the individual lending. It is an optimization model inspired from Bhole and Ogden (2010), Tedeschi (2006) and Ghatak (1999) by introducing a new measure flow dictated by a limited number of lending cycles.

2. Model with Cycles of Refinancing

In order to avoid the problem of non-payment due to the default strategy, and avoid permanent exclusion of services of the MFI, Tedeschi (2006) proposed a model based on the incentive of borrowers used by Besley and Coate (1995) and Ghatak (1999).

This incentive model takes the form of refinancing the project. Indeed the defaulting borrower will be excluded from refinancing only for some period.

This type of contract is a game to an infinite period where each player (here the MFI and the Borrower) aims to maximize his expected profits.

The individual contract offered by the MFI will be held for each lending cycle as follows:

Step 1: The MFI offers a contract characterized by clauses in the contract (lending size, interest rate, mode of repayment, maturity).

Step 2: The borrower invests the lending in a project.

Step 3: The borrower decides to fully or partially respect its contract .

Depending on the respect of the contract signed to each cycle, the MFI will choose or not to refinance the project under the conditions that we will specify in the following paragraph.

So the borrower, in order to reaches his State of SRM, he must first successfully complete all lending cycles, but in case of no repayment of a maximal amount fixed by the MFI, this one definitely cancel the refinancing process because the borrower has failed to being SRM.

In the following we are interested to optimize the utility of the borrower and the number of lending cycles which allow the borrower to be SRM.

For i = 0,1, …, k let $P(\theta = H_i)=\vartheta$ where H_i is the return generated by the project at cycle *i* at which borrower must pay an amount $R_{I,i}$, and let R_i the amount it may decide to repay the MFIs hope that $R_{I,i} - R_i$ is zero. In practice $R_i \geq R_{I,i}$.

2.1. Process of Refinancing

In the initial cycle, the borrower receives I unit lending with unit cost *z* (in this paper we consider *z* Fixed).

If the borrower repays R_0 such that $R_0 < R_{1,0}$, then the MFI receives message (implicit signal) from borrower which showed that he deployed effort to respect the clauses in his contract, the response from the MFI will be in the form of incentive that gives the borrower to access to the next cycle number 1 but under conditions as sanctions.

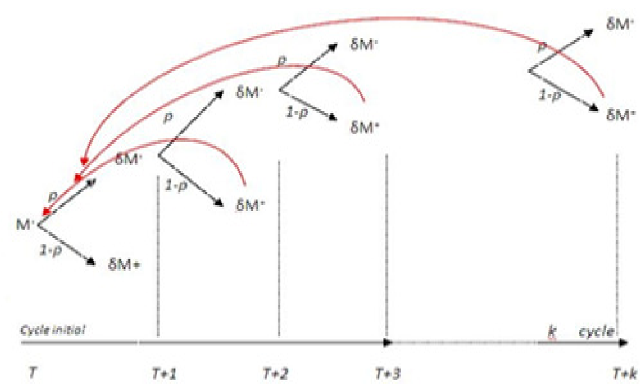
Passing to cycle 1, the MFI gives to borrower lending with other clauses. Indeed, if again $R_1 < R_{I,1} + R_{I,0} - R_0$, then the MFI gives to borrower, as a reward, access to the next cycle but with new clauses containing more sanctions than the previous cycle.

The figure1 below shows the model that we propose and which generates the following decision variables:

- p_k is the conditional probability to cancel the refinancing if the borrower succeed his project and refused to pay during the k^{nd} cycle $R_{I,k}$ is the amount that borrower must repay in cycle k without counting the rest to pay from initial cycle to cycle $k-1$.
- ϑ_k and H_k are related by the equation $P(\text{income} = H_k) = \vartheta_k$ where H_k is a profit generated by the project.
- α_k a strictly increasing sequence of the interval] 0, 1 [, such that $(1 + \alpha_k)^1$ is updated and include new interest rate $(1+\alpha_k)I$, is the amount that the borrower will receive when he succeed in cycle k-1.

This application takes two forms, a reward for that a borrower continues to refinance and maintain his project and learn the techniques of risk management, but also a form of sanction generating more charges.

Figure 1: After an exclusion period of length T, MFI can to determine a number of cycles which the total amount is repaid



One cycle $k \in \mathbb{N}$ is characterized by the variables $(P_k, R_{I,k}, \vartheta_k, H_k, \alpha_k)$

2.2. Proposed Model

The proposed model implicitly generates the optimal number k of cycles, allowing the MFI to confirm that the borrower became SRM, and also minimizing risk posed by loss due to the rest of amounts to repay.

Let M^- the expected utility function of the initial cycle, and let M^+ the expected utility function if the borrower has refinancing, (Figure1).

for simplicity, we assume that ϑ_k and H_k are constants throughout the process, and pi = pj for i, j =1, …, k.

Given the discount profit we have:

$$M^- = \delta^T (1-p) \frac{1-(\delta p)^k}{1-\delta p} M^+ = \delta^T (1-p) \sum_{i=0}^{k} (\delta^i p^i) M^+ \tag{1}$$

Using the same notation at section 2.1., we have:

$$R_k = \sum_{i=0}^{k-1} (r_i) - R_{I,k} \tag{2}$$

Where $r_i = R_{I,k-} R_i$, $r_i \geq 0$.

In cycle k the total expected utility function is written:

$$M^+ = \vartheta(H - R_k - \delta M^+) + (1-\vartheta)\delta M^- \tag{3}$$

Combining (1) and (3) we can write M^+, as follows:

$$M^{+}=\frac{\vartheta(H-R_k)}{1-\vartheta\delta+\delta^{T+1}(1-\vartheta)(1-p)\dfrac{1-(\delta p)^k}{1-\delta p}} \tag{4}$$

therefore:

$$M^{-}=\frac{\vartheta(1-p)(1-(\delta p)^k)\delta^{T}(H-R_k)}{(1-\vartheta\delta)(1-\delta p)+(1-\vartheta)(1-p)(1-(\delta p)^k)\delta^{T+1}} \tag{5}$$

The goal is to maximize M^+ defined by (4).

The constraints

1. In order to incite the borrower to repay the lending in case of success i.e. the prevention of strategic default. The incentive constraint of the project is:

   $R_k \leq \delta\,(M^+-M^-)$ (6)

   Denoting

   $A = (1-p)(1-(\delta p)^k), \lambda = \delta^T(1-\vartheta), \mu = \delta\vartheta$,

the constraint (6) becomes:

$$R_k \leq \frac{\mu(1-\delta p-\lambda A)H}{1+(1-\mu)(1-\delta p)-\delta\lambda A} \tag{7}$$

2. Participation: the cumulative amount should not exceed the income generated by borrower project:

   $Rk \leq \forall k\in$ IN (8)

3. The nullity of profit because the MFI is a non-profit organization:

   $\vartheta R_k \leq (1+a_k)I+z\ \forall k\in$ IN (9)

   Or

$$R_k \leq \frac{(1+\alpha_k)I+z}{\vartheta} \tag{10}$$

Where z is the unit cost associated in lending unit.

Given δ, α_k et z, the optimization problem of the MFI is to find (p, R, k) such:

$$(p_1)\begin{cases} \text{Max}_{q,p,RI} M^{+}=\dfrac{\vartheta(H-R_k)}{1-\vartheta\delta+\delta^{T+1}(1-\vartheta)(1-p)\dfrac{1-(\delta p)^k}{1-\delta p}} \\ R_k \leq \dfrac{\mu(1-\delta p-\lambda(1-p)(1-(\delta p)^k)H}{1+(1-\mu)(1-\delta p)-\delta\lambda(1-p)(1-\delta p)^k)} & (c1) \\ R_k \leq H & (c2) \\ R_k \geq \dfrac{(1+\alpha_k)I+z}{\vartheta} & (c3) \\ 0\leq p\leq 1 & (c4) \end{cases}$$

Remark: Practically the MFI would not be able to continue to renew the lending to the borrower, so k cannot tend to infinity.

3. Numerical Solution

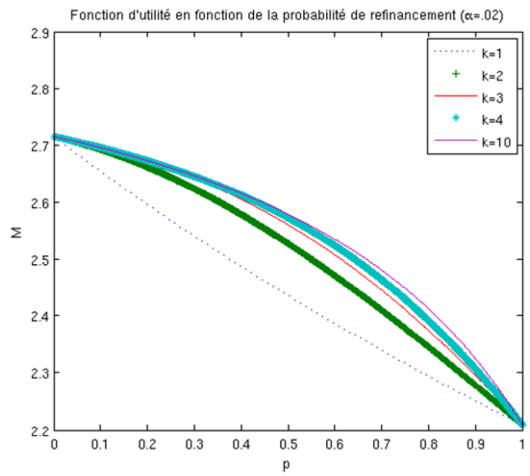
Before completely solve the problem (P_1), we note that the function M^+ is decreasing as a function of P see figure below.

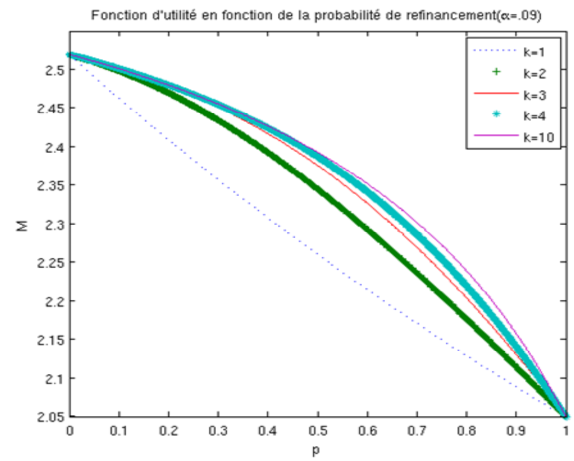
According Bhole and Ogden (2010), the optimal value of R_k will always take the same form:

$$R_k=\frac{(1+\alpha_k)I+z}{\vartheta} \tag{11}$$

as α_k is strictly increasing then it is the same for R_k since both are linearly dependent and positives. With each new refinancing the MFI taking an increasing risk than the previous cycle, but also tries to have the cumulative unpaid amount. (Graph 1 and 2):

Graph 1 and 2: M = f9p- for $\alpha = 0.09$ and z = 0.02., where $\delta = 0.09$., H = 2.5, T = 1.5 T, V = 0.8, I = 1, z = 0.01, k=2





Changing Variables

By replacing the expression of R_k the constraint (c1) is written,

$$A_1(1-x) \geq \delta A_2(1-x^k)(\delta-x)+(1+\alpha_k)I$$

Denoting: $x = \delta p, A_1 = \mu H - (1-\mu)R_k et A_2 = \lambda(\frac{\mu H)}{\delta} - R_k)$

Thus the problem (P_1) is written

$$(p_1)\begin{cases} Max_{q,p,RI} M^+ = \dfrac{\vartheta H - (1+\alpha_k)I}{1-\vartheta\delta+\delta^{T+1}(1-\vartheta)(1-\frac{x}{\delta})\frac{1-x^k}{1-x}} \\ \quad s/c \\ A_1(1-x) \geq \delta A_2(1-x^k)(\delta - x) + (1+\alpha_k)I \qquad (c'1) \\ 0 \leq x \leq \delta \qquad (c'2) \end{cases}$$

Table: Numerical values given by (P_2), for I = 1, $\vartheta = 0.7$. H = 2.5 and z = 0.01

α	**1%**			**2%**			**3%**		
δ	**0.7**	**0.85**	**0.9**	**0.75**	**0.85**	**0.9**	**0.8**	**0.85**	**0.9**
p	0.9	0.21	0.09	0.62	0.24	0.12	0.43	0.27	0.15
k	1.1275	1.9971	2.0008	1.9909	1.9962	1.9995	1.9927	1.9953	1.9985
M^+	2.1429	4.2857	6.4286	2.5143	4.1905	6.2857	3.0714	4.0952	6.1429

The obtained values are the exclusion probability of the future lending P, the cycle's number k, and M^+ the wellness function of the borrower. These values are generated by the new model where α is a reward for the borrower to refinance and continue the maintaining of his project and learn risk management techniques. It can be also used as a sanction.

The maximum of M^+ is 6.4186 and It is obtained for $\alpha = 1\%$ and $\delta = 0.9$. This value is obtained after two cycles and the corresponding exclusion probability is 9%. This probability is relatively low. Moreover, the wellness function of the borrower decreases depending on α. It means that the MFI should encourage borrower by refinancing every two cycles with less severe penalties that are reduced on α.

4. Conclusion

The model that we have developed will allow to the MFI to monitor individual lending in terms of cycle's number and the cumulative amount to be repaid from the initial cycle adding wellness of the utility function of the borrower. As the group lending with shared responsibility contract is the most adopted by MFIs. In order to obtain a more realistic model, we will generalize the model developed in this work to the contract for group lending, considering a conflict of interest within the group lending.

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